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JUNIOR HIGH SCHOOL MATHEMATICS
By William Ledley Vosburgh, Frederick William Gentleman and Jasper O. Hassler.

JUNIOR HIGH SCHOOL MATHEMATICS

FIRST COURSE

BY

WILLIAM LEDLEY VOSBURGH

HEAD OF DEPARTMENT OF MATHEMATICS
TEACHERS COLLEGE, CITY OF BOSTON

FREDERICK WILLIAM GENTLEMAN

LATE JUNIOR MASTER, DEPARTMENT OF MATHEMATICS
THE MECHANIC ARTS HIGH SCHOOL, BOSTON

AND

JASPER O. HASSLER

PROFESSOR OF MATHEMATICS
UNIVERSITY OF OKLAHOMA

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PREFACE

In revising the course in mathematics for the junior high school, the authors have found that the former outlines of material have justified themselves on the basis of the teaching outcomes; that is, the knowledge of mathematics acquired by the pupils and the ability to use this mathematics in practical situations. As in the earlier edition, the following points have been kept in mind:

1. That instruction in mathematics in the seventh school year of necessity must begin at the point which standard tests have shown that the pupil has reached as a result of the work in arithmetic of the first six grades.

2. That such tests show that the pupil has acquired an automatic mastery of certain number facts, a knowledge of how the four processes with integers and fractions are performed, and a limited acquaintance with the facts and relations of the commonly used denominate units.

3. That the course in mathematics in the junior high school should be of such content that it will bring the pupil in contact with adult activities which lend themselves to mathematical interpretation and will afford him an opportunity for the exercise of his mathematical powers.

4. That the course in mathematics should be so administered, that the pupil becomes habituated to the standards of the business world; that is, since the computer must assume the responsibility for the correctness of his computation, he must always, by check or by estimate or by both, satisfy himself of the correctness of his work before it leaves his hands.

The principal additions or changes which have been made in the revision of the First Course are these: (1) The history of the development of certain topics is told informally for the purpose of making the pupil realize the vital connection that mathematics has with the development of civilization; (2) graded drill tables and proficiency tests have been included; (3) the amount of material included has been expanded from six chapters to eight chapters; (4) graphic methods are used earlier in connection with the records on proficiency tests; (5) several extended problem situations have been included; (6) there is more problem material throughout, particularly on the business applications of percentage; (7) questions for review have been added at the end of each chapter.

The attention of teachers is again directed to these features of the First Course: (1) the nature and amount of practice material for the improvement of the arithmetical ability of pupils; (2) the simple forms of checking the work in addition and subtraction of numbers; (3) the emphasis placed on the estimate of the result and on rational methods of locating the decimal point in multiplication and division; (4) the early and continued use of the equation as a mathematical tool; (5) the natural introduction of the idea of ratio and of per cent as a special kind of ratio; (6) the rational development of measurement and the extension of the decimal idea to this field by the use of the protractor and decimalized ruler; (7) the use of graphical methods in the interpretation and representation of number data; (8) the restriction of the treatment of percentage to the simple direct applications; (9) the emphasis on reasoning and common sense throughout the work in mensuration; (10) the selection, character, and amount of applied work presented.

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JUNIOR HIGH SCHOOL MATHEMATICS

FIRST COURSE

CHAPTER I

REVIEW OF NUMBERS

I. Reading and Writing Integers

1. Do you know that ages ago people did not know how to count? No one could count up to ten, for nobody had made any names for numbers. The people were not civilized as we are—they did not even have a system of reading and writing. Finally some one thought of making a symbol for a man, probably like this (), a symbol for sunrise (), and symbols for other things they knew about. From this grew up a kind of picture writing. The American Indians used such picture symbols.

If a man sent a message concerning two men or three men, he drew two symbols () or three symbols () to represent the men. Suppose he wanted to say that some event had happened three times. He soon learned to write three straight marks (III) to stand for three as it does to-day on the faces of some clocks, as chapter numbers in books, and in various other places.

Imagine a boy who lived long ago who had caught five fishes. He tries to count them. He touches them one by one, saying some words like our one, two, three, and

then he stops; but there are more to count. He cannot count them. He may say, "One, two, three, *many.*" If some one asks him how many fishes he has, he may say, "Many," or he may hold up five fingers. Did you ever answer a similar question in that manner?

Number names were finally devised up to five, and the notion of hand was connected with five. In the Russian language the words for "fist" and "five" differ by one letter and sound very much alike. The Kiowas and kindred tribes in Oklahoma used a picture of a hand to stand for five. People who counted by "hands" (fives) learned to say "hand and two" for seven and "hand and three" for eight, etc., just as we have "ten and three" represented in the symbol 13. But they could only count up to "five hands and five," or 30. Any greater number was "many." Next came the idea of using both hands. The reason our number system is based on ten is probably because we have ten fingers. Suppose we should have had eight fingers, or twelve!

People who wore no shoes sometimes used 20 as the base for their system of counting. Some South American Indians had the same word for twenty that they used for "one Indian" (twenty fingers and toes). An Indian with 21 horses would tell the number by saying, "One on the other Indian" (meaning as many as one Indian has fingers and toes, and one of another Indian's fingers). That our own ancestors often counted by twenties is shown by the various meanings of the word *score* in English and by similar words in other European languages.

As man became more civilized and language symbols began to represent syllables and sounds instead of words, the symbols for numbers became simpler. The Egyptians

in the Nile valley 5000 years ago used | for one, ∩ for ten, and wrote ∩ ∩ ||| for 24.

It would be quite a task to learn to multiply and divide numbers written like those already mentioned. As people began to trade more and more, they needed to use numbers for commercial computations. A computing machine called the *abacus* was invented. The two diagrams below (Figs. 1, 2) illustrate the principle. On the abacus are rods (or grooves) along which slide "buttons," or markers, whose values depend on the rod on which they are placed. Those in the first row (on the right)

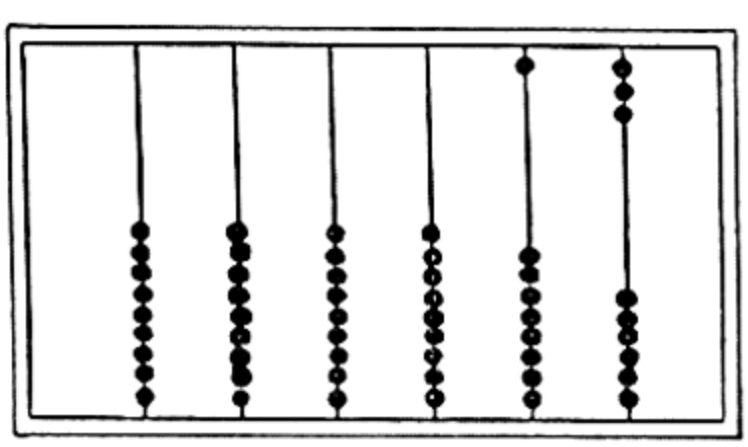


FIG. 1.

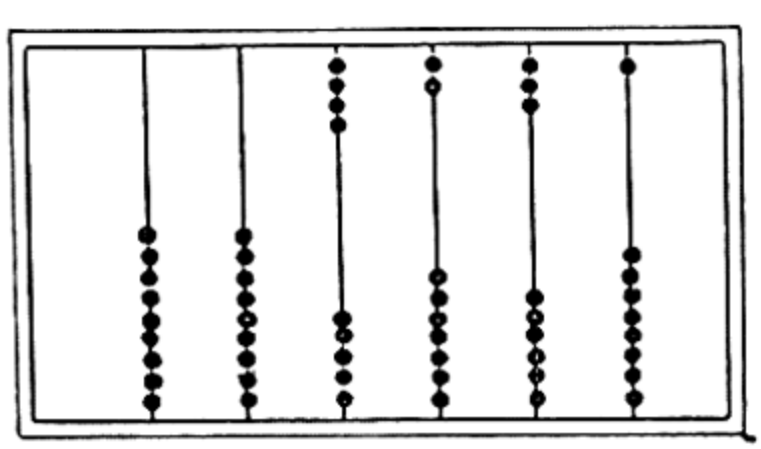


FIG. 2.

represent ones, those in the second row tens, etc., each button having the value of ten buttons of the next row to the right. If a man counted up to nine he would slide over nine markers of the first row. If he wanted to count, say thirteen, he would, when he was ready to put the tenth marker out, slide back all nine of the first row and put out one of the next row to represent ten and put out three in the first row (see Fig. 1). The adjustment of markers in Fig. 2 represents the number 4231. The Chinese still use a form of the abacus in some of the calculations that they make and are said to be very skillful in the use of it.

The ancient Greeks (2500 years ago) used the letters of their alphabet for numbers. Various other symbols were used, and finally people came to use 1, 2, 3, 4, 5, 6, 7, 8, 9. These symbols are called *digits* or *figures* or *numerals*. For centuries after knowing and using these nine symbols people used the abacus for computation. You wonder why? Because it was convenient to have the rods determine the different values of the markers since there was no way to write the digits so that anyone else would know whether one stood for tens, hundreds, or units. They did not know how to write 305 as we do because the symbol 0, which we call *zero* or *cipher* or *naught*, was unknown. On the abacus if no marker of value ten was to be used, the upper end of the rod was simply left empty. To write it as 3 5 would confuse it with thirty-five. The Hindus (in Asia) about 1400 years ago conceived the idea of using a symbol to mean "nothing" and completed the system of numbers as we use them. The Arabs learned this method from the Hindus and passed it on to our European ancestors. Hence we call it the *Hindu-Arabic system*.

2. When two or more of these symbols are written together, the number value then represented by each symbol depends also upon the place it occupies. Thus, using 2 and 5, you can write the numbers twenty-five (25) or fifty-two (52). In the first number 2, by the place it occupies, represents two *tens*, 5 represents five *units;* in the second number 5 represents five *tens*, 2 represents two *units*. Using 2, 3, and 5 together you can write six numbers: 235, 253, 523, 532, 325, 352.

Noting the place that each figure occupies in each of the above numbers, tell the number that it represents.

3. The number value that a figure may express by reason of the place (or *order*) that it may occupy in a number is spoken of as its *place value;* the value that it denotes individually is its *figure value.*

4. When 0 (zero) is used with other symbols it may help to *place* them and so express number value; as 230, 320, 302, 203, 023, 032.

Read each of the above numbers, and tell in which of them the 0 helps to express number values.

Since the 0 serves no purpose in writing the last two numbers, it is generally not written. When the zero is written first, as in the last two numbers, it shows that no omissions or oversights have occurred; that is, that the first figure is really a zero. Read the following numbers:

560, 506, 605, 650, 2650, 2600, 2065, 2605, 2005.

Do not use *and* in reading whole numbers (integers).

Read these numbers again as you give telephone numbers. In computing and in checking, numbers are often read in this way.

5. For convenience in reading numbers of four or more figures, in the Hindu-Arabic system the figures generally are separated by commas into groups of threes, called ***periods.*** Thus we write 243,605; 3,865,327; etc.

The table on page 6 shows the arrangement of *periods* and *orders.* Beginning at the foot of this table, see whether you can read each number in the table as follows:

Two thousand five hundred sixty-seven; two thousand five hundred; two thousand sixty-seven; etc.

Note that in reading a number each period is named after the number in that period has been read *except in units' period.*

BILLIONS (Orders)			MILLIONS (Orders)			THOUSANDS (Orders)			UNITS (Orders)		
hundreds	tens	units	hundreds	tens	units	hundreds	tens	units	hundreds	tens	units
8	7	6	9	8	0	0	0	0	2	0	7
1	2	3	7	0	6	0	0	5	0	2	9
	3	1	2	1	1	3	0	0	4	0	2
	5	4	4	5	0	0	3	0	0	0	3
		2	5	6	0	4	0	0	0	3	0
		3	4	0	7	9	6	3	0	0	0
			1	0	2	3	0	0	2	0	0
			2	5	0	3	0	2	0	5	6
				8	0	0	2	0	0	6	0
				2	5	4	3	0	0	0	4
				3	0	7	0	5	0	0	6
				7	0	8	1	0	0	0	2
					2	6	7	0	3	0	
					9	5	0	5	9	0	
					6	6	7	7	7	6	
						5	9	4	3	7	
						9	5	3	4	7	
						2	0	0	0	9	
						2	0	0	9	0	
						2	5	0	0	9	
						2	5	6	0	9	
						5	2	2	5	0	
						4	3	3	7	6	
						3	4	7	3	6	
							2	0	7	0	
							2	0	0	7	
							2	0	0	0	
							2	0	6	7	
							2	5	0	0	
							2	5	6	7	

6. Summary: (1) There are ten separate symbols used in our system (the *Hindu-Arabic system*) of writing numbers. These symbols are called figures, numerals, or digits.

(2) Each figure has a *figure* value and a *place* value.

Thus, the figure value of zero is naught; but a zero that occupies a place in a number may be very important indeed.

(3) The place each figure occupies in a number is called an *order*.

(4) In large numbers the figures are grouped by threes; these groups of threes are called *periods*.

(5) The names of the orders (single places) in each period are limited to three. Beginning at the right they are: Units (or ones), tens, and hundreds. (See the table, page 6.)

(6) The names of the periods are units (or ones), thousands, millions, billions, trillions, etc. (See the table, page 6.)

A million is a thousand thousand. A billion is a thousand million. A trillion is a thousand billion.

(7) The relation between successive orders is that of tens; that is, the number value of a figure is multiplied by ten when it is moved from one order to the next order on its left, and it is divided by ten when it is moved to the next order on its right.

(8) The two essential things are:

(*a*) To be able to read and write any number of three figures.

(*b*) To know the names of the periods.

Practice in these essentials is afforded by the exercises that follow.

EXERCISES

A. *Write in figures each of the following numbers:*

1. Two hundred forty-two.
2. Two hundred forty.
3. Two hundred four.
4. Five hundred fifty.
5. Five hundred five.
6. Two thousand two hundred six.
7. Two thousand twenty-six.
8. Five thousand two hundred ten.
9. Five thousand two hundred one.
10. Two thousand five hundred one.
11. Two thousand five hundred ten.
12. Two thousand two.
13. Thirty thousand three hundred thirty.
14. Thirty thousand three hundred three.
15. Thirty thousand three.
16. Fifty-six thousand nine.
17. Twenty thousand twenty-two.
18. Twenty thousand two.
19. Three hundred six thousand two hundred twelve.
20. Three hundred thousand seven hundred eighty.
21. Three hundred thousand eight hundred seventy.
22. Three hundred thousand eight hundred seven.
23. Seventy thousand seven hundred seven.
24. Seven thousand twenty.
25. Three million four hundred three thousand three hundred four.
26. Three million twenty-two thousand four.
27. Ten million one thousand one hundred.

B. *Write in words each of the following numbers:*

1. 526
2. 562
3. 420
4. 402
5. 1254
6. 2154
7. 3145
8. 4230
9. 4203
10. 4023
11. 7020
12. 7002
13. 7200
14. 6060
15. 6006
16. 6600
17. 5902
18. 9502
19. 9009
20. 9900
21. 23,006
22. 20,306
23. 20,036
24. 20,006
25. 50,075
26. 50,705
27. 50,005
28. 210,432
29. 203,400
30. 200,980
31. 300,073
32. 902,001
33. 927,006
34. 3,020,052
35. 2,201,002
36. 6,067,308
37. 60,006,006
38. 60,600,060
39. 320,000,200
40. 3,000,780,500

C. *Express in words and also in figures each of the following numbers:*

1. The number of feet in a mile.
2. The number of pounds in a ton.
3. The number of square feet in an acre.
4. The number of minutes in a school day of five hours.
5. The population of your city or town.
6. The population of your state; of the United States.
7. The area of your state in square miles.
8. The number of pupils in your city or town.

II. Reading and Writing Decimals

7. A *decimal fraction*, or a *decimal*, is a fraction whose denominator is 10, 100, or 1000, etc. The denominator is not written but is expressed by the decimal point (.) placed in front of the numerator. Thus,

.3 expresses $\frac{3}{10}$.
.23 expresses $\frac{23}{100}$.
.234 expresses $\frac{234}{1000}$.

8. To express some decimals, one or more zeros must be inserted between the decimal point and the numerator. Thus,

$\frac{3}{100}$ is written .03
$\frac{3}{1000}$ is written .003
$\frac{23}{1000}$ is written .023

9. An integer and a decimal together, called a ***mixed decimal***, express a ***mixed number***. Thus,

3.2 expresses $3\frac{2}{10}$.

10. *To read a decimal*, read it as a whole number and then give it the name of the last decimal place. Thus,

.206 is read two hundred six *thousandths*.

.0322 is read three hundred twenty-two *ten-thousandths*.

To read a mixed decimal, read the integral part (the whole number) as usual, use *and* at the decimal point, and then read the decimal. Thus,

7.206 is read seven *and* two hundred six *thousandths*.

212.2581 is read two hundred twelve *and* two thousand five hundred eighty-one *ten-thousandths*.

11. The table on page 11 shows the arrangement of *orders* in our decimal system. Beginning at the foot of the table, see whether you can read each number in the table.

Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths
2	5	0	0	0	0	.	0	0	0	5
	1	3	5	0	0	.	0	0	2	5
		5	0	0	2	.	2	0	1	5
		3	4	0	0	.	0	5	0	8
				9	0	.	0	4	3	6
					0	.	1	2	4	5
				4	0	.	0	0	4	
			2	5	0	.	2	5	0	
			2	5	0	.	0	2	5	
			5	0	0	.	0	6	7	
			3	0	2	.	2	0	3	
			2	0	3	.	3	0	2	
				1	8	.	0	5	9	
					7	.	0	1	0	
					3	.	0	0	2	
					8	.	0	3	0	
					9	.	4	0	8	
					4	.	1	3	0	
					9	.	1	3	6	
					3	.	9	8	5	
					8	.	0	8	4	
					5	.	0	4	3	
					1	.	2	3	0	
					2	.	2	3	4	
					3	.	2	3	2	
					5	.	2	0	6	
					0	.	8	0	9	
				1	2	.	8	6	8	
					2	.	6	0		
					3	.	0	6		

EXERCISES

Read each of the following numbers:

1. 0.2
2. 0.02
3. 0.023
4. 0.020
5. 0.0026
6. 0.260
7. 0.500
8. 0.12
9. 0.21
10. 0.012
11. 0.021
12. 0.120
13. 0.210
14. 0.312
15. 0.213
16. 0.123
17. 0.302
18. 0.203
19. 0.230
20. 0.049
21. 0.409
22. 0.490
23. 0.940
24. 0.400
25. 0.900
26. 3.2
27. 8.50
28. 7.120
29. 14.02
30. 5.026
31. 5.202
32. 6.305
33. 9.210
34. 8.206
35. 7.012
36. 30.03
37. 30.30
38. 303.3
39. 3.030
40. 33.003
41. 26.62
42. 26.26
43. 62.26
44. 2.626
45. 226.06
46. 50.05
47. 55.50
48. 550.05
49. 505.55
50. 5.005
51. 121.12
52. 112.21
53. 100.01
54. 200.02
55. 1.001
56. 256.203
57. 102.302
58. 200.031
59. 2002.02
60. 600.06
61. 60.006
62. 60.60
63. 600.006
64. 6.0060
65. 43.043
66. 30.030
67. 5002.002
68. 3010.010
69. 300.30
70. 800.8
71. 700.0007
72. 400.0004
73. 4006.300
74. 4050.003
75. 32.0040
76. 70.0070
77. 201.1020
78. 5005.0090
79. 9008.0007
80. 7070.6007
81. 6600.0056
82. 4001.1004
83. 6005.5006
84. 8000.0008

12. *To write a decimal in figures:*

(1) Write the numerator as you write any whole number.

(2) Place the units' figure of this whole number in the decimal order given.

(3) Insert zeros, if needed, between the decimal point and the numerator.

Thus, to write in figures twenty-six *thousandths*,

(1) Write 26.

(2) Place 6 in *thousandth's* order.

(3) Insert one zero.

It then appears as .026.

EXERCISES

Write in figures the following:

1. Seven tenths.
2. Nine tenths.
3. Ten tenths, or one and no tenths.
4. Thirteen tenths, or one and three tenths.
5. Nineteen tenths.
6. Six hundredths.
7. Twenty-five hundredths.
8. Thirty-two hundredths.
9. One hundred twenty-five hundredths.
10. Three hundred forty-six hundredths.
11. Twenty-eight thousandths.
12. Three hundred two thousandths.
13. Nine thousandths.
14. Ninety thousandths.
15. One hundred twenty thousandths.
16. One hundred and twenty thousandths.

17. Three hundred and two thousandths.
18. Three hundred two thousandths.
19. Forty-seven and six thousandths.
20. Six hundred and six tenths.
21. Six hundred six thousandths.
22. Six and six hundredths.
23. Six and sixty thousandths.
24. Two hundred and two hundredths.
25. Five thousand and five thousandths.
26. One hundred and one thousandth.
27. Two hundred and two hundred-thousandths.
28. Two hundred two hundred-thousandths.
29. Six hundred and six hundred-thousandths.
30. Three hundred three and three thousandths.
31. Nine hundred and nine hundred-thousandths.
32. Nine hundred ninety and ninety-nine thousandths.

13. In business practice and in scientific work the pure decimal .24 is commonly written 0.24; that is, the absence of any integral part is emphasized by writing 0 in units' place.

Read each of the following quantities·

$0.56; 0.2 in.; 0.72 ft.; 0.125 in.; 0.80 ft.

Uses of zero: Zero is used in the following different ways for the purpose stated:

(1) To fill up vacant places in numbers, as 702; 1060; 200,007; 6.007.

(2) To show that there are no oversights or omissions, as $0.96; 0.3 in.; Tel. Main 0395.

(3) To designate a particular order or unit in our decimal system, as 70% = .70; 23.0%; 8.0 in.

III. The Roman System

14. On the Italian Peninsula in southern Europe for several centuries before and after the beginning of the Christian era there lived a sturdy race of people called Romans, from their capital city, Rome, which was situated exactly where the Italian city of Rome stands to-day. The Romans are noted for their conquests, the great power of their government, and their laws. Their language (Latin) furnishes the foundation for many of our English words. Their number system, however, was very crude. Like the ancient Greeks (page 4) they used letters for numbers, but according to a better scheme.

In the ***Roman system***, seven separate symbols (capital letters) are used:

I = one.	C = one hundred.
V = five.	D = five hundred.
X = ten.	M = one thousand.
L = fifty.	

These letters are combined as follows.

(1) If a letter is repeated, its value is repeated; for example,

$$\text{II} = 2, \quad \text{XX} = 20, \quad \text{CCC} = 300, \quad \text{MM} = 2000.$$

(2) If a letter is placed before one of greater value, its value is taken from that of the greater, for example,

$$\text{IV} = 4, \quad \text{XC} = 90, \quad \text{IC} = 99, \quad \text{XL} = 40.$$

(3) If a letter is placed after one of greater value, its value is added to that of the greater; for example,

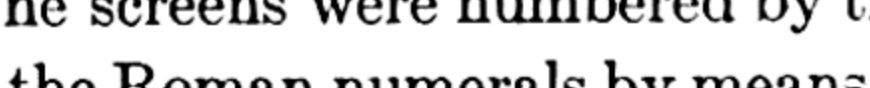
$$\text{VII} = 7, \quad \text{CXV} = 115.$$

Note. When a letter is placed between two of greater value, rule (2) takes precedence over rule (3); that is, the

value of the letter of least value is subtracted from the value of the letter following it, or it is subtracted from the sum of the values of the two letters between which it is placed; for example, XIX = 19, CIV = 104.

(4) If a bar is placed over a letter, its value is increased a thousandfold; for example, $\bar{V} = 5000$.

15. Many tribes of North American Indians used a system of numbers similar to the Roman, but it was limited to two main symbols. They used | for one and + for ten. They also had a special symbol ⊕ for one dollar. A line of a certain Indian's personal account with a store runs like this:



To the Indian this meant, "Thirty pounds of venison at ten cents a pound is worth three dollars." He would write 32 as +++||. The Romans wrote it XXXII.

The Roman system of numbers was so cumbersome that they used the abacus for most of their calculations. Their abacus had grooves along which they moved pebbles. The Latin word for pebble is *calculus*. In this you see the origin of our word *calculate*.

The Roman system is used to-day in certain exceptional cases, such as chapter headings, dates on monuments, on buildings, etc. It is a very convenient system of numbering when for some reason it becomes difficult to make curved symbols. The symbols for all the numbers less than 100 can be made with straight lines. For example, one of the authors of this book wanted to number the window screens of his house in some way so that they could be removed, painted, and put back in their proper places. The screens were numbered by the simple method of making the Roman numerals by means of the sharp edge

of a chisel driven into the wood, making each time a straight mark half an inch long. The letters I, V, X could be made easily, whereas it would have been impossible to make the symbols 2, 3, 8, and some others.

EXERCISES

Read the following numbers:

1. XC	6. XCIV	11. CXXXVI
2. XXIV	7. CCLII	12. MDCXX
3. LII	8. MCII	13. MDCCLXXVI
4. XLII	9. MD	14. MCMXVI
5. XLIV	10. $\overline{\text{M}}$	15. MDCLXVI

Write the following numbers in the Roman system:

16. 36	20. 142	24. 1812
17. 47	21. 196	25. 1900
18. 51	22. 1660	26. 1923
19. 90	23. 1781	27. 20,000

28. The year of your birth.

29. The year the armistice was signed when the Allies had defeated Germany and Austria.

30. The year Columbus discovered America.

QUESTIONS FOR REVIEW

1. Using the Indian methods, in how many different ways could you say that you had caught eight fishes?

2. How many horses did a South American Indian have when he said that he had "one hand on the other Indian"?

3. Draw a picture of an abacus and show how the number 1301 would be written on it.

4. How many different symbols are used in our system of writing numbers, and what are they called?

5. Illustrate what is meant by the place value of a digit or figure in a number.

6. See whether you can name the orders, beginning at the decimal point and going to the left for seven places; beginning at the decimal point and going to the right for four places.

7. What is a decimal fraction? A mixed decimal?

8. When should we use "and" in reading numbers?

9. Read the following numbers as *tenths:* 1.7; 2.3.

10. Read the following numbers as *hundredths:* 0.12; 1.12; 2.03; 3.40; 3.00.

11. Write in figures each of the following numbers: five thousand five; five thousand fifty; forty thousand four hundred four; forty thousand four.

12. Write in figures each of the following numbers: four hundred and forty hundredths; four hundred forty hundredths; four and forty hundredths.

13. Write in figures each of the following numbers: six thousand and six thousandths; six thousand six; six and six thousandths.

14. Write in figures each of the following numbers: five hundred and five hundred thousandths; five hundred five hundred-thousandths; five hundred thousandths.

15. Fill in the missing numbers, and write them in at least two different ways:

A dozen = ? ones.	A square foot = ? square inches.
A pound = ? ounces.	A gallon = ? quarts.
A yard = ? feet.	A square yard = ? square feet.
An hour = ? minutes.	A cubic foot = ? cubic inches.
A quart = ? pints.	A mile = ? feet.

16. If you know of a monument or building on which the date of its erection is written in the Roman system, show the class how it is written and read the date for them.

CHAPTER II

THE FOUR FUNDAMENTAL PROCESSES
INTEGERS AND DECIMALS

16. Addition. You know the process of addition. The numbers added are called *addends*. The *sum*, or *amount*, is the result obtained.

More is expected of you now than to know merely *how* to add. You must learn to add *accurately* and *with greater speed. Speed without accuracy is worth little.* Therefore accuracy is to be desired most of all. However, efficient business demands also reasonable speed. To help you become more proficient, use Table I on page 20 for daily practice in the addition combinations.

Read through the first line, giving only results, as 6, 12, 9, 7, 17, 5, 13, etc. Then the second line, etc. See whether, by practice, you can reduce the time for reading each line to about ten seconds.

Use the table also according to the following plan. Let everyone cover his table with a sheet of paper. When the teacher says, "Begin," slip the paper down with the upper edge just below some line assigned by the teacher. Write the results along the edge, stopping promptly when the teacher says, "Stop," at the end of *ten* seconds. If you finish a line within the ten seconds, begin another. Check the results, and record the number you have correct. The incorrect results are worth nothing.

If the number of correct answers written by one pupil on six successive days are, in order, 10, 11, 12, 12, 14, 15, it would show improvement. If the records of another pupil

are 13, 12, 12, 13, 11, 12, which pupil has made the better record?

TABLE I

5	6	7	4	8	4	5	9	2	0	5	9	3	0	2
1	6	2	3	9	1	8	7	6	3	7	8	6	4	1
9	8	5	4	1	6	5	4	7	3	0	8	1	6	3
2	7	6	4	3	8	5	2	9	1	9	5	2	7	4
5	9	7	6	8	2	1	7	0	5	9	0	6	3	8
2	9	1	3	4	9	4	8	6	3	4	5	1	7	2
4	1	7	6	2	4	6	7	8	9	8	7	5	0	3
8	1	5	9	3	5	2	7	1	3	6	4	9	1	2
6	1	2	3	8	4	8	7	2	1	0	1	9	3	4
4	9	7	5	0	9	8	3	5	6	2	5	6	8	7
4	1	3	9	2	9	8	2	5	7	7	3	6	1	2
6	7	3	5	8	1	3	2	4	6	0	9	5	8	4

Repeated use of this table may make you familiar with the answers. In Chapter VIII there is another table.

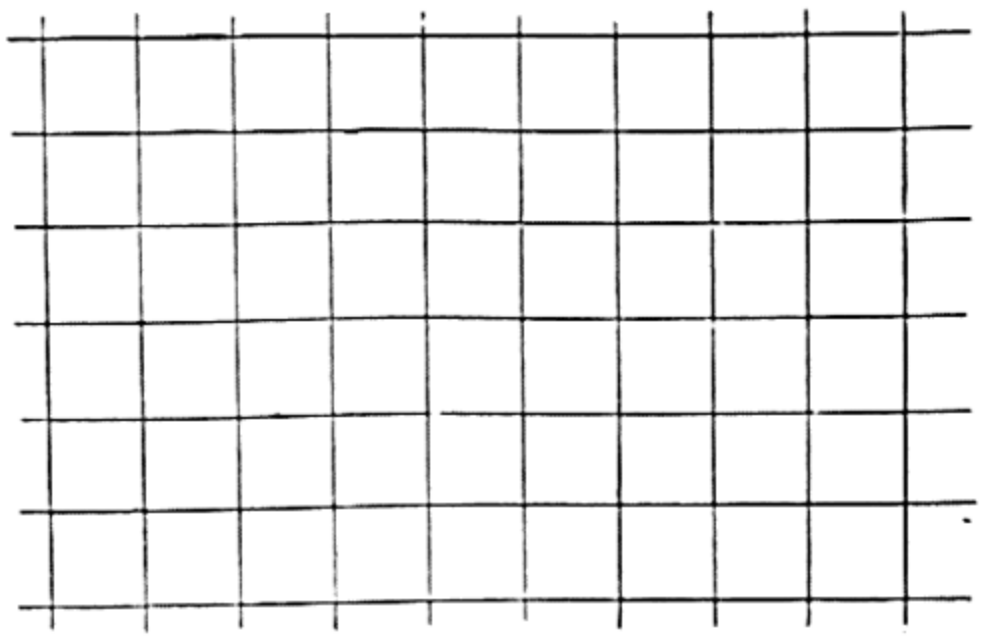


FIG. 3.

17. Graphical Pictures of Progress in Proficiency. On a sheet of ruled paper draw a series of cross-ruled lines making right angles with the lines on the paper. Make them as far apart as the lines on the paper. Your paper will then look like the diagram in Fig. 3.

Begin at the second vertical line on the left at the bottom of the sheet and write the numbers 1, 2, 3, 4, 5, 6 on successive vertical lines. Let these numbers represent the order in which the tests were taken. On the line marked 1, measure up ten spaces from the bottom and shade heavily that part of the vertical line. [See Fig. 4.] Repeat

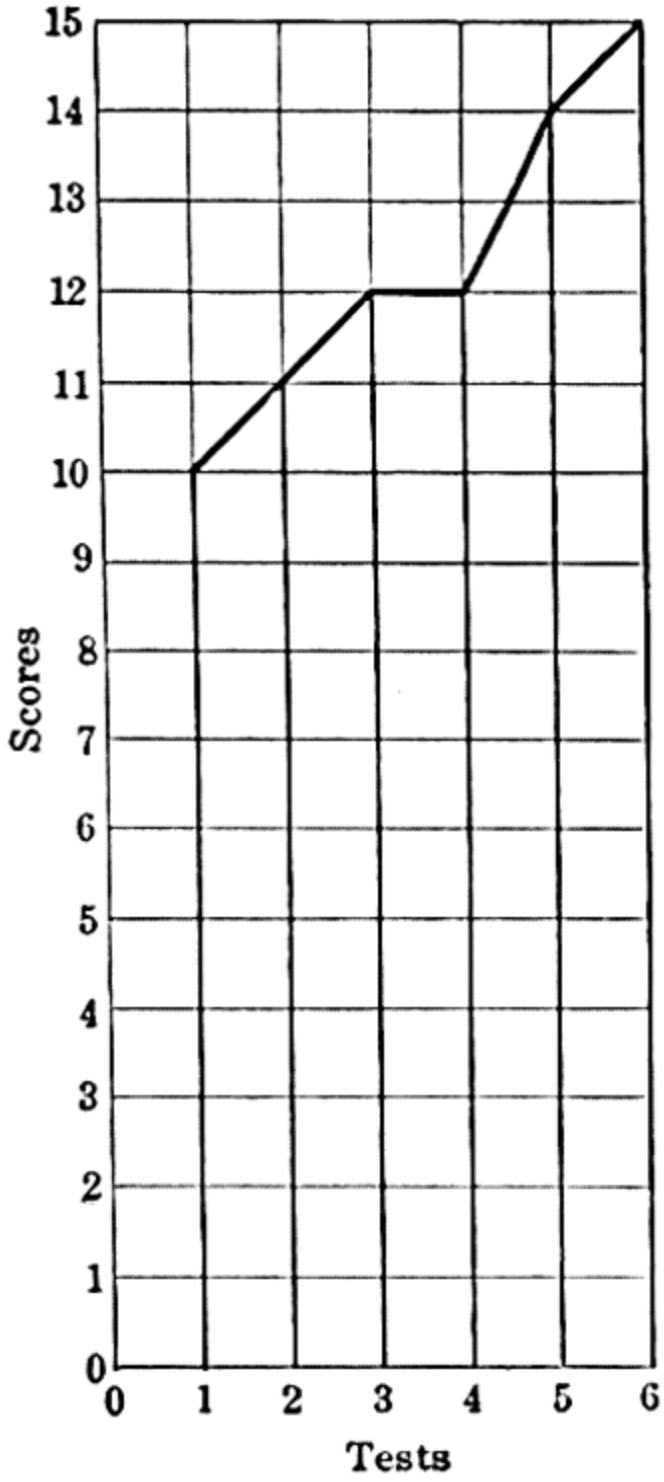


FIG. 4.

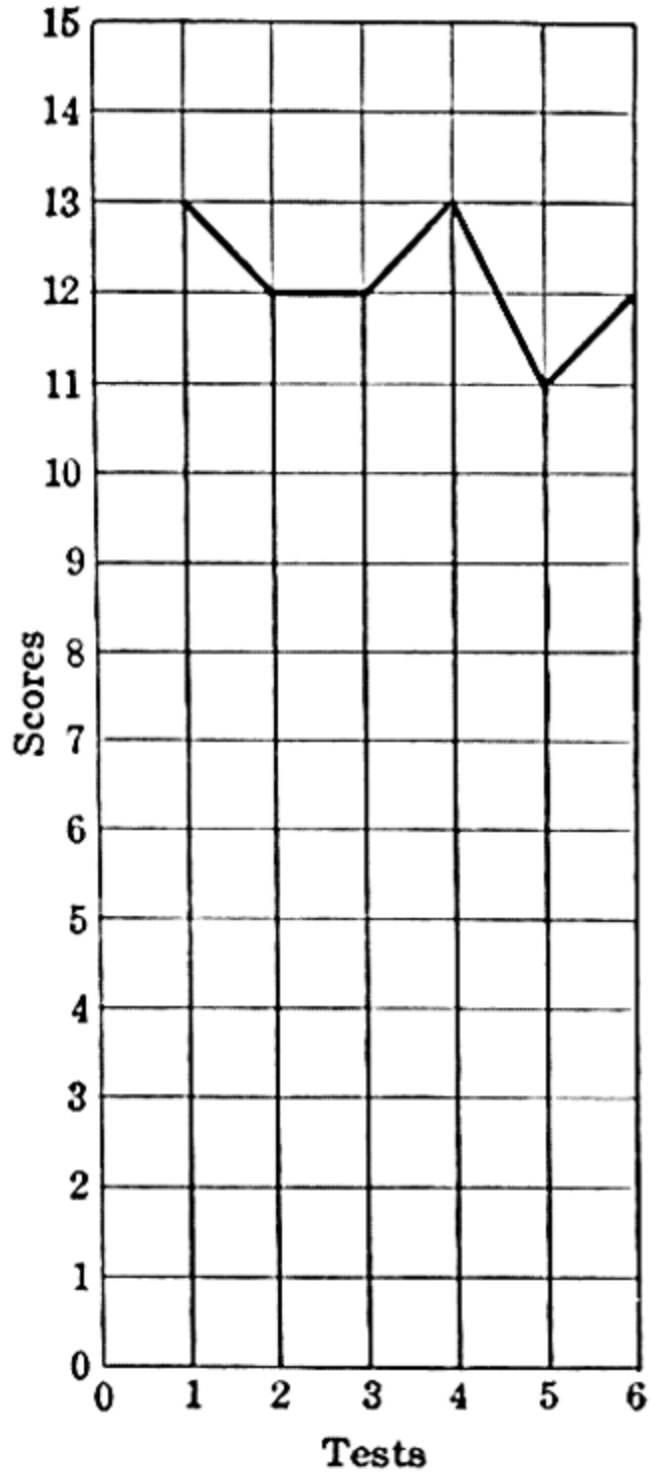


FIG. 5.

this for the first 11 spaces on the vertical line marked 2, 12 on the third, 12 on the fourth, 14 on the fifth, and 15 on the sixth. The lengths of these shaded pieces represent the results of the first pupil mentioned in §16 on the six tests. If we join the upper ends of the shaded lines we have a broken line which "climbs" upward.

If we measure in a similar manner on another sheet of paper the results of the second pupil mentioned in §16, we have a diagram as in Fig. 5. These broken lines are called *graphs*. Graphs are being used more and more in business to picture to the eye records of various kinds, such as records of proficiency like these. Have you noticed any graphs in a newspaper or magazine?

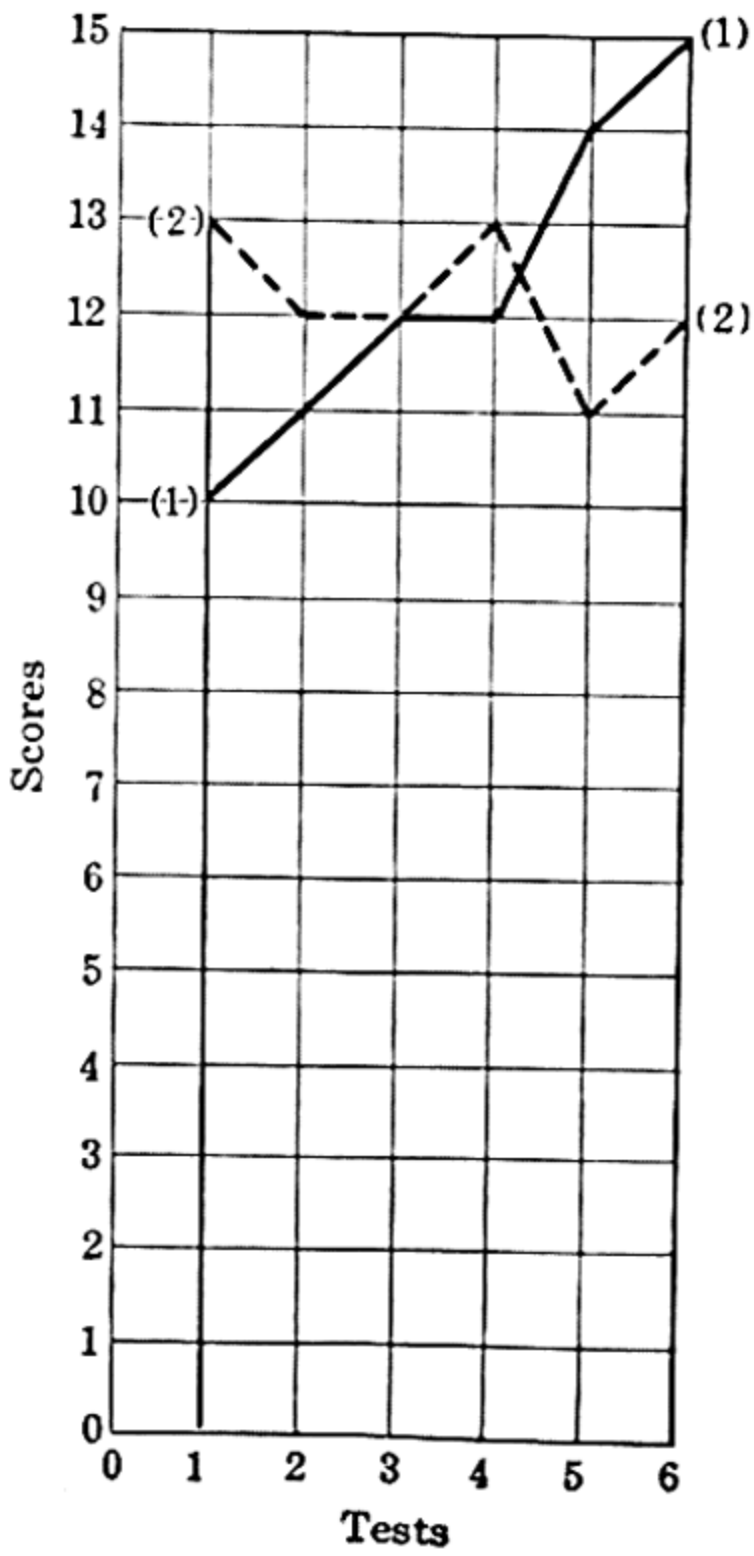


FIG. 6.

Now draw both graphs on the same sheet of paper, omitting the shading of the lines (which may be confusing when you combine them). The effect is shown in Fig. 6. Which one of the pupils' records would you prefer to have? Which pupil do you think did not practice to improve his ability?

In each of the tests of this chapter that you take repeatedly to test your improvement in addition, you should keep a record of your scores and draw a graph showing your progress. Try to better your record day by day so that the graph of your progress will "climb" upward as it moves to the right across the paper.

The following exercises will show you other subjects for graphs. You should begin to be more interested in the graphs that may be found in newspapers and magazines.

EXERCISES

1. Mary had the following records for correct answers on a certain repeated test in addition: 10, 10, 11, 12, 11, 12, 14, 15. Draw a graph showing her progress.

2. John's records on the same tests were 9, 10, 12, 11, 10, 11, 12. Draw the graph of John's records.

3. Five boys in a class made on the same series of tests the following individual scores:

I. 8, 7, 10, 9, 10, 9, 8, 11
II. 10, 12, 11, 13, 13, 14, 14, 15
III. 7, 8, 8, 9, 9, 11, 11, 13
IV. 12, 13, 13, 14, 14, 15, 16, 17
V. 8, 10, 8, 10, 9, 11, 11, 14

Draw graphs of the individual scores on separate sheets of paper.

4. Get the total scores for the boys of Problem 3 on each test and divide each total by 5. This will give you the average score. Draw a graph of this average score. Is it a record of which the boys should be proud?

5. Draw a graph of the attendance record of your class for two weeks (using for your successive numbers the number present day by day). What should be the nature of this graph if the attendance is perfect?

6. Get the average score for the boys in your class on the addition of the combination of Table I for ten seconds. Also get the average score for the girls. Compare the two averages. Let some one keep a record of these averages as succeeding tests are taken on this table, and draw the graphs of the average records. Which group wins, the boys or the girls? Probably you will want to continue this contest in the drills on the tables that follow. Or you may prefer to choose teams in some other way.

TABLE II. Daily practice in reading through this table will help you to become more rapid and accurate in adding columns of figures.

Read through the first line, always adding *upward* and giving only the results, as 12, 52, 42, 22, etc. Then the second line, etc., until you have finished all the lines.

Keep a record of your time in seconds for each line. See whether, by practice, you can reduce the time for reading each line to about *twelve* seconds.

TABLE II

7	7	7	7	7	7	7	7	7	7
5	45	35	15	25	95	75	55	85	65
8	8	8	8	8	8	8	8	8	8
6	26	16	46	36	86	66	96	76	56
9	9	9	9	9	9	9	9	9	9
7	37	27	17	47	77	57	87	67	97
6	6	6	6	6	6	6	6	6	6
9	19	49	39	29	59	89	69	99	79
8	8	8	8	8	8	8	8	8	8
15	17	18	16	19	37	28	46	39	55
7	7	7	7	7	7	7	7	7	7
17	19	15	18	16	38	45	57	29	66
5	9	7	8	9	5	8	6	9	7
28	38	99	97	88	98	99	98	89	97

TABLE III. Daily practice in reading through this table will also help you to become more rapid and accurate in adding columns of figures. Use this table in the same way that you have used Table II. Adding upward, say 10, 19; 12, 20; etc.

TABLE III

9	8	6	7	5	7	8	9	5	4
5	6	7	8	9	4	4	4	4	4
5	6	7	8	9	5	6	7	8	9
7	8	9	5	6	9	5	6	7	8
6	7	8	9	5	7	8	9	5	6
5	6	7	8	9	5	6	7	8	9
7	8	9	5	6	8	9	5	6	7
8	9	5	6	7	9	5	6	7	8
5	6	7	8	9	5	6	7	8	9
7	8	9	5	6	8	9	5	6	7
8	9	5	6	7	9	5	6	7	8
15	26	37	48	59	25	36	47	58	69
9	5	6	7	8	5	6	7	8	9
5	6	7	8	9	6	7	8	9	5
35	46	57	68	79	45	56	67	78	89
6	7	8	9	5	7	8	9	5	6
7	8	9	5	6	4	4	4	4	4
55	66	77	88	99	65	76	87	98	99

TABLE IV. Use this table (and the supplementary exercises of the same kind in Chapter VIII) in the following manner: Start with some given row in the table assigned by the teacher. Beginning when your teacher gives the signal, add as many exercises as you can in *three* minutes. Write the answers on a slip of paper. If you finish more than one row before time is called, fold down the paper and continue with the next row. Keep a record of your scores as you try this day by day, even after you are many pages further along in the book, and note your improvement. Plot a graph showing the change in your scores. Perhaps two teams from the class will enjoy a contest for the higher average score.

TABLE IV

629	358	294	185	360	295	871
812	937	836	564	726	850	267
285	937	175	836	493	567	428
928	717	936	639	728	748	326
283	545	274	384	595	284	785
647	567	475	685	365	546	503
756	364	635	254	513	241	357
968	586	847	475	635	364	842
253	524	374	746	587	869	666
260	158	370	257	397	258	545
842	951	602	632	555	902	383
565	476	846	243	835	254	777

18. To meet the demands of the business world to-day you must form the habit of checking your work.

To check your work you should do two things:

(1) Leave a record of the successive steps of the computation.

(2) Always go over the computation a second time in some different manner, or order, if possible.

In writing down numbers to be added it is necessary that you make the figures legible and write figures of the same order in the same vertical columns. This is especially important in work that is to be read by others, or which is to be kept as a record.

As addition is the process most frequently used, the habit of checking is most important here.

A simple method of recording the successive steps (column sums) in addition is shown below.

Also a second addition of each column (downward) is shown, and the checking up of the column sum by the check mark (√).

EXAMPLE 1

8786	(Record of work)
5674	3 3√
8789	4 0√
6587	3 8√
8967	3 8√
38803	

EXPLANATION. The sum of units' column is 33; 3 is carried to the next column and the sum 40 is set down under the 33; 4 is carried to the next column and the

sum 38 is set down under the 40; 3 is carried to the next column and the sum 38 is set down under the 38. In the "Record of work" the sum of two columns is 403, of three 3803, of all 38,803.

CHECK. If the columns are added upward, check by adding downward. After each column has been added downward and found to be correct, put a check mark (√) by the sum for that column.

This method of recording the successive sums of columns makes it possible, in checking another person's work, to compare the answers obtained at the end of the corresponding columns as the work progresses.

EXAMPLE 2

$5,462.28	
379.87	5 4√
4,687.66	4 7√
278.89	5 7√
86.38	6 3√
5,787.76	3 9√
2,576.67	2 4√
25,678.23	4√
$44,937.74	

NOTE. The decimal point is not regarded in the check record.

While many business offices now use adding machines to add such columns, a great many sums must be done without the machines. In any case the work of machines must be checked frequently.

EXERCISES

Copy and find the sums of each of the following examples. Leave a record of the column sums as shown in the preceding examples. Check each answer by the method shown above.

1.	2.	3.	4.	5.	6.
675	974	887	987	577	969
894	596	698	878	799	888
567	678	789	799	889	797

7.	8.	9.	10.	11.	12.
345	567	768	892	367	478
567	458	269	195	357	598
168	207	526	269	476	379
468	674	377	467	589	497

13.	14.	15.	16.	17.	18.
256	366	274	197	269	369
579	375	437	388	365	479
907	158	357	455	797	499
396	775	463	277	469	365
666	475	144	320	568	779

19.	20.	21.	22.	23.	24.
356	435	456	379	479	559
536	767	819	267	637	589
276	207	316	468	557	385
215	227	234	245	236	256
326	345	327	365	324	354
189	197	195	194	196	198

25.	26.	27.	28.	29.	30.
345	356	358	379	369	389
467	487	497	479	488	469
549	597	534	557	573	592
694	685	643	671	694	685
736	753	789	768	795	789
847	852	875	894	859	877
947	996	966	987	969	958

31.	32.	33.	34.	35.	36.
4.67	5.90	$4.29	57.9	35.7	$7.93
4.78	5.87	3.25	57.2	38.0	6.85
4.80	5.89	2.36	57.6	38.1	9.23
4.64	5.89	4.76	56.8	37.6	10.43
4.68	5.92	2.35	56.7	38 2	5.75
4.76	5.85	4.16	59.2	36.7	7.17
4.85	5.70	6.26	57.5	36.9	13.62
4.69	5.84	4.25	58.3	37.7	8.28

37.	38.	39.	40.	41.	42.
56.75	89.43	5.845	$36.78	$67.54	$123.55
57.18	88.88	5.367	42.25	74.28	95.67
58.06	90.67	5.444	75.45	92.56	7.74
57.66	89.23	5.777	28.93	76.54	105.25
58.26	88.77	5.767	99.07	25.94	19.52
57.62	89.67	5.676	43.78	98.89	6.25
58.22	88.76	5.646	34.87	89.98	154.52
57.66	87.97	5.464	86.78	28.89	54.45

43.	44.	45.	46.
659,789	6,857	695,498	987,456
675	479.584	765,489	576,897
7,598	687,489	987,654	768,498
76,958	95,795	456,789	895,765
957,869	48	856,497	684,976
697	87,654	774,596	567,895
89	558,789	698,975	456,789

47.	48.	49.	50.
453.78	$2,678.84	54.763	39.562
476.96	1,837.77	57.879	43.278
437.96	947.34	63.769	33.574
468.59	3,057.49	47.895	50.578
457.49	2,198.49	50.777	46.556
482.47	2,567.98	61.858	38.995
467.78	948.84	49.945	47.333
446.65	1,395.67	55.789	49.946

51.	52.	53.	54.
72,308	84,165	612,384	4,615
6,213	125,004	4,126	27
73	269	27,000	67,903
1,208	1,300	5,269	729
5,799	875,695	879,548	894
89,746	5,678	897	6,945
559,876	756	684,578	587
788	69,587	485	876,694
6,546	6,958	87,699	945,889
645,678	87,569	694,877	95,674

19. Subtraction. As a preparatory exercise, tell what number should be added to the lower number to give the upper number.

10	10	10	10	10	10	10	10	10	10
5	6	1	3	4	8	2	9	0	7
11	11	11	11	11	11	11	11	11	11
4	8	1	5	9	0	2	6	3	7
12	12	12	12	12	12	12	12	12	12
6	7	5	0	3	8	1	4	9	2
13	13	13	13	13	13	13	13	13	13
7	9	1	3	6	8	0	2	5	4
14	14	14	14	14	14	14	14	14	14
8	5	3	9	0	1	4	7	2	6
15	15	15	15	15	15	15	15	15	15
0	4	9	1	7	6	2	5	8	3
16	16	16	16	16	16	16	16	16	16
2	0	6	9	5	4	1	7	3	8

The word *subtraction* suggests that something is to be *taken away from* something else.

The ***subtrahend*** is the number which is taken away or subtracted.

The ***minuend*** is the number from which you take the subtrahend.

The result of the subtraction is called the ***remainder,*** or ***difference.***

These names were invented when the main idea in the process of subtraction was to "take away" something. The "addition" or "shop" method of subtraction is suggested in this book. Instead of asking the question, "What is the remainder when 5 is subtracted from 9?" we ask, "What number added to 5 will give 9?"

This method has the following advantages over other methods:

(1) It is a self-checking method since that number is found which if added to the subtrahend will give the minuend.

(2) There is no "borrowing" difficulty.

(3) It is the universal method of "making change" in the business world.

EXAMPLE 1

WORK	EXPLANATION OF WORK
√√ √√√	
56,134	6 and ? are 14, write **8**, carry 1 to 5
23,456	6 and ? are 13, write **7**, carry 1 to 4
32,678	5 and ? are 11, write **6**, carry 1 to 3
	4 and ? are 6, write **2**, (no carrying)
	2 and ? are 5, write **3**

EXPLANATION OF CHECK

Add upward the answer to the subtrahend. Thus:

8 + 6 = 14, put check (√) over 4; carry 1 to 7

8 + 5 = 13, put check (√) over 3; carry 1 to 6

7 + 4 = 11, put check (√) over 1; carry 1 to 2

3 + 3 = 6, put check (√) over 6; (nothing to carry)

3 + 2 = 5, put check (√) over 5

This method of checking is independent of the method of subtraction used. Identical errors are practically impossible.

EXERCISES

Copy and find the difference, or remainder, in each of the following exercises. Check each answer by adding it to the subtrahend to see whether this sum gives the minuend.

1.	2.	3.	4.	5.
207,568	$3,086,120	$250,000	75,129	108,977
137,243	1,536,376	187,441	43,694	33,117

6.	7.	8.	9.	10.
124,723	232,743	$350.	947,362	210,001
66,069	21,079	187.31	75.129	64,577

From the first amount take the second in each of the following exercises:

11. $201.10, $87.67
12. $700.03, $157.48
13. $3100.02, $837.16
14. $82,102.00, $71,191.17

Take the first number from the second in each of the following exercises:

15. 272.3, 1201.1
16. 394.02, 1002.2
17. 738.16, 2100.6
18. 56.25, 3103.52
19. 867.72, 2130
20. 1027.2, 1410.96

Find the difference between each of the following pairs of numbers:

21. 66.3 and 43.7
22. 160.5 and 124.87
23. 1523.4 and 75.88
24. 2001 and 782.6
25. 32.563 and 325.63
26. 0.0246 and 0.0057

Find the excess of the first number over the second in each of the following exercises:

27. 1215.76 over 87.876
28. 2103.17 over 897.29
29. $1567.13 over $999.87
30. $2,122,000 over $787,026
31. 3,010,001 over 1,357,973
32. 0.0206 over 0.00678

TABLE V. Use this drill table as you did Table IV in addition. Try to write as many correct answers as you can in *three* minutes. Keep a record of your scores and draw the graph. Also average the scores of teams chosen in some manner and draw the graphs for the team averages. The team with the poorest graph should strive to improve until its graph crosses the others, as it "climbs" higher on the cross-ruled paper.

TABLE V

9243	8274	9683	9102	8295	8967
3786	2847	5348	1576	2837	2536
8310	7395	7892	8341	9372	7961
3768	3947	2317	2987	2804	4168
9012	8294	6132	8793	9264	7980
2856	2847	2845	2047	1907	2146
6102	7593	6981	9534	8260	7962
1937	2468	3702	1056	3095	4937
8423	9238	6872	9204	8275	6975
2698	2958	3417	3078	2859	1058
8360	9163	8347	9103	7159	8234
1987	1046	3877	1357	3099	3654
8642	9173	7204	9753	7234	6021
2678	2777	4055	7359	5106	3578

20. Multiplication. The *multiplicand* is the number to be multiplied.

The *multiplier* is the number by which you multiply.

The result of the multiplication is called the *product.*

In order to renew your acquaintance with the multiplication table, try reading the products in the following table. How many can you read in *ten* seconds?

TABLE VI

2	9	3	7	5	8	4	6	5	6	4	7
6	3	7	4	3	5	6	9	7	8	4	6
7	4	6	5	6	4	8	5	7	3	9	2
2	8	5	4	3	9	7	6	8	4	7	8
2	6	3	9	4	7	3	6	5	8	2	9
9	7	8	2	2	7	2	6	5	9	5	4
9	2	8	5	6	3	7	4	9	3	8	2
9	2	3	9	4	6	9	3	6	3	8	7
8	2	2	5	2	4	3	5	6	7	7	9
2	9	3	8	4	7	5	6	2	9	3	8
9	7	5	3	8	6	4	2	4	6	5	7
5	3	2	5	2	2	5	4	7	7	8	5
3	8	4	2	3	8	9	8	7	8	9	4
9	6	5	3	9	4	5	6	5	4	8	5
6	9	7	5	8	6	9	6	8	5	8	7
4	2	3	6	5	3	3	7	5	7	3	6
2	4	4	2	5	3	6	4	2	4	3	2
7	5	8	6	4	4	8	9	8	2	6	3

A multiplication table is used for the purpose of saving time in computation. Its value to us depends entirely on how alert we are in giving the products as in the table on the preceding page. If we want to know, for example, how many 7 times 8 are, we can either write down 8 seven times and add or write from memory the product as we have learned it in the table. If you find it necessary to stop to make sure of such a product, the table is not worth much to you; you should learn it better.

The Greeks used such a multiplication table about 2000 years ago. In spite of their cumbersome method of using letters for the number values, they improved their computation a great deal by means of the table. The ancient Egyptians 4000 years ago had no multiplication table. To multiply two numbers, they would double one and halve the other until a product was reached. While the method involved a more complex operation than the following simple illustration, it will show in general what the method was:

$$8 \times 13 = 4 \times 26 = 2 \times 52 = 1 \times 104 = 104.$$

In multiplication, especially of decimals, you should make a round-number *estimate* of your product *before* performing the work. You should write down this estimate when you make it and submit it as a part of the record of your work.

You should use this estimate to check up the location of the decimal point in your product, thus avoiding serious, as well as absurd, blunders.

For an accurate check in multiplication, you should do the work a second time, using the old multiplicand as the new multiplier and the old multiplier as the new multiplicand.

PROBLEM. *Find in pounds the weight of* 82.6 *cu. ft. of water.* [NOTE. One cubic foot of water weighs 62.5 pounds.]

METHOD (*a*)		METHOD (*b*)
62.5		62.5
82.6	ESTIMATE	82.6
3750		5000.
1250	$80 \times 60 = 4800$	125.0
5000		37.50
5162.50		5162.50

EXPLANATION OF METHOD (*a*). We form the partial products by multiplying the multiplicand by each digit in the multiplier in order, proceeding from *right* to *left*. These partial products are set down under each other, each being advanced one place to the left; and they are then added to form the final product.

In the final product we locate the decimal point by the following rule: The number of digits to the right of the decimal point is equal to the total number of digits to the right of the decimal points in both multiplicand and multiplier.

EXPLANATION OF METHOD (*b*). In multiplying by method (*b*) we use the figures of the multiplier in order from *left* to *right* instead of from right to left as usual.

To obtain the first partial product the figure 8 of the multiplier is used first. Now 8 here represents 8 *tens*, that is, 80; so we place the decimal point in this first partial product after the 5000 (estimate was 4800).

To obtain the next partial product, we multiply by 2, placing the figure first obtained in this partial product one column to the *right* of the preceding product.

To obtain the last partial product, we multiply by 6, placing the figure first obtained in this partial product again one column to the *right* of the preceding product.

In this method we locate the decimal point by estimate and common sense early in the work, that is, in the first partial product. Carelessness and errors in its location in the final product are thereby prevented.

ACCURATE CHECK

METHOD (*a*)	ESTIMATE	METHOD (*b*)
82.6		82.6
62.5		62.5
4130	$60 \times 80 = 4800$	4956.
1652		165.2
4956		41.30
5162.50		5162.50

EXERCISES

Carry out each of the following indicated multiplications:

1. $12 \times \$21.50$	11. 324×3.142	21. 7.60×285
2. $15 \times \$24.74$	12. 532×39.37	22. 5.32×483
3. $22 \times \$31.13$	13. 643×62.48	23. 2.37×327
4. $26 \times \$23.32$	14. 962×3.142	24. 7.59×964
5. $34 \times \$38.83$	15. 870×3.375	25. 2.63×7358
6. $30 \times \$42.55$	16. 458×8.63	26. 3.07×56.78
7. $60 \times \$54.45$	17. $786 \times 48{:}83$	27. 5.02×159.24
8. $72 \times \$83.65$	18. 290×39.83	28. 6.23×354.85
9. $75 \times \$64.46$	19. 608×62.479	29. 0.587×385.4
10. $58 \times \$30.05$	20. 803×2150.42	30. 0.382×56.37

Use the following table as you have used Tables IV and V and try to get as many products in *five* minutes as possible.

TABLE VII

8395	2847	9315	3706	1648	9207
35	82	91	84	73	56
2974	9136	5074	1697	3083	1479
48	39	27	64	92	85
8064	2538	6721	3574	7026	4813
86	26	38	49	72	18
8563	2947	8406	1738	9036	2468
36	43	47	54	24	52
2475	9068	3648	2815	6931	8742
23	36	59	75	79	62
8694	2463	8795	3054	6578	2351
18	65	91	47	73	56
3775	8258	4906	4676	9216	5083
23	45	67	89	42	86
2975	6843	5903	2574	8765	4987
83	72	19	56	34	76
9630	1357	8642	2946	3078	2573
59	81	37	84	92	65

21. Division. The ***dividend*** is the number to be measured or divided.

The ***divisor*** is the number by which the dividend is measured or divided.

The result of the division is the ***quotient.***

EXAMPLE 1. *Divide* 37,647 *by* 89.

WORK

$$89\overline{)37647}\quad 423$$

356

204

178

267

267

CHECK

423

89

3807

3384

37647

It is clear that division is really the reverse of multiplication; the process is also the reverse of the process for multiplication.

For an accurate check in division multiply the quotient by the divisor and add the remainder, if any, to this product. The result should be the dividend.

EXAMPLE 2. *Divide* 451.36 *by* 6.2.

METHOD (*a*)

For this example multiply both numbers by 10 to make the divisor a whole number; then divide as in integers. $451.36 \div 6.2$ becomes $4513.6 \div 62$.

Place the decimal point in the quotient over the decimal point in the dividend.

METHOD (*b*)

Estimate the size of the answer before dividing, as $450 \div 6 =$ about 70.

Place the decimal point in the quotient by comparison with the estimate.

WORK

$$\begin{array}{r} 72.8 \\ 62\overline{)4513.6} \\ 434 \\ \hline 173 \\ 124 \\ \hline 496 \\ 496 \\ \hline \end{array}$$

CHECK

$$\begin{array}{r} 72.8 \\ 6.2 \\ \hline 1456 \\ 4368 \\ \hline 451.36 \end{array}$$

WORK

$$\begin{array}{r} 72.8 \\ 6.2\overline{)451.36} \\ 434 \\ \hline 173 \\ 124 \\ \hline 496 \\ 496 \\ \hline \end{array}$$

CHECK

$$\begin{array}{r} 72.8 \\ 6.2 \\ \hline 436.8 \\ 14.56 \\ \hline 451.36 \end{array}$$

EXERCISES

Carry out each of the following indicated divisions to *four* figures:

1. $42\overline{)294,966}$
2. $54\overline{)378,540}$
3. $68\overline{)136,920}$
4. $73\overline{)928,654}$
5. $93\overline{)837,372}$
6. $82\overline{)166,330}$
7. $71\overline{)568,279}$
8. $91\overline{)822,134}$
9. $89\overline{)2911.32}$
10. $98\overline{)88,437}$
11. $89\overline{)359.09}$
12. $66\overline{)71.982}$
13. $9.9\overline{)812.655}$
14. $5.3\overline{)162.579}$
15. $8.7\overline{)43,365.3}$
16. $322\overline{)877,846}$
17. $224\overline{)498,263}$
18. $42.1\overline{)97,978.3}$
19. $2.13\overline{)489.90}$
20. $3.15\overline{)367.86}$
21. $4.21\overline{)127.42}$
22. $7.20\overline{)367.86}$
23. $11.4\overline{)47.138}$
24. $4.71\overline{)12.742}$
25. $26.4\overline{)154.764}$
26. $38.5\overline{)243.726}$
27. $74.8\overline{)369.457}$
28. $59.2\overline{)784.436}$
29. $8.62\overline{)1943.936}$
30. $7.61\overline{)1846.363}$
31. $6.32\overline{)182.03}$
32. $741\overline{)21.134}$
33. $852\overline{)72,260}$
34. $354\overline{)769,234}$
35. $236\overline{)702,263}$
36. $3.21\overline{)7022.63}$
37. $0.36\overline{)7.6342}$
38. $0.43\overline{)8.8264}$
39. $0.85\overline{)9.7843}$

TABLE VIII. Use the table below to test your proficiency as you have used the other tables. If the divisor is not contained in the dividend a whole number of times, the remainder after the final subtraction should be written. If there is no remainder, zero should be written. Let your time limit be *five* minutes.

TABLE VIII

37)6,493	53)25,378	72)13,752	57)9,375	29)4,758
65)11,365	76)9,037	46)7,105	83)37,594	19)4,680
95)57,286	88)9,783	27)6,456	18)4,567	39)8,642
48)8,866	62)24,637	94)67,380	35)6,990	47)5,928
58)25,894	61)36,207	73)9,472	84)22,675	77)33,445
36)7,105	54)37,294	28)8,642	82)48,294	43)9,734
81)23,478	17)6,281	59)8,137	67)28,943	71)49,780
26)23,002	68)42,406	93)27,621	78)38,006	97)48,975
66)45,682	89)76,543	38)37,506	98)89,768	47)36,021

Perhaps you would be interested to choose several teams in your class (the simplest arrangement would be for each row of pupils as they are seated to make up a team) and have an arithmetic field meet.

Make a test on each of Tables I, IV, V, VI, VII, VIII an "event." Get the average score of each team and give 5 points for a "first," 3 for a "second," and 1 for a "third." The team securing the greatest total number of points wins.

MISCELLANEOUS PROBLEMS

1. An Automobile Road Map gives the following distances:

Boston to Springfield	94.7 miles
Springfield to Pittsfield	56.0 miles
Pittsfield to Albany	36.1 miles
Albany to Utica	95.0 miles
Utica to Syracuse	61.8 miles
Syracuse to Buffalo	152.0 miles
Buffalo to Niagara Falls	25.9 miles

(*a*) Find the distance from Boston to Pittsfield.
(*b*) Find the distance from Boston to Albany.
(*c*) Find the distance from Boston to Syracuse.
(*d*) Find the distance from Boston to Buffalo.
(*e*) Find the distance from Boston to Niagara Falls.
(*f*) Find the distance from Boston to Utica.
(*g*) Find the distance from Springfield to Syracuse.
(*h*) Find the distance from Springfield to Buffalo.
(*i*) Find the distance from Albany to Buffalo.
(*j*) Estimating the expense of operating an automobile at eight cents per mile, find the cost of trips between cities named in (*a*) to (*i*).
(*k*) If an automobile carries seven persons from Boston to Niagara Falls and each of them pays for his share of the cost of running the car, how much should each one pay for the trip?

2. At $1.86 a bushel, what is the value of a carload of 425 bushels of potatoes?

3. At $1.08 a bushel, what is the value of a carload of 1225 bushels of wheat?

4. In a class of 18 pupils the records on a certain test were: 97, 95, 93, 92, 86, 86, 85, 82, 78, 78, 74, 74, 72, 71, 67, 62, 57, 55. Find the total and the average.

[NOTE. To find the average divide the total by the number of pupils.]

5. In a class of 18 pupils on a spelling test of 50 words the records were: 50, 49, 48, 48, 43, 42, 42, 39, 37, 36, 35, 35, 32, 31, 30, 28, 28, 27. Find the total and the average.

6. The following were the records of a class in a speed test of one minute in addition: 53, 51, 48, 48, 48, 46, 46, 46, 46, 46, 43, 43, 40, 40, 40, 35, 35, 31, 25. How many pupils took this test? Find the average record.

7. A's salary for each of the last six years has been: $1800, $1900, $2340, $2484, $2628, $2772. Find his average yearly salary for this term of years.

8. The heights of 8 boys in a class are as follows: 5.9 ft., 5.8 ft., 5.8 ft., 5.7 ft., 5.4 ft., 5.0 ft., 4.9 ft., 4.7 ft. Find the average height.

9. The heights of 10 boys in a class are as follows: 5 ft. 1 in., 5 ft., 4 ft. 11 in., 4 ft. 10 in., 4 ft. 10 in., 4 ft. 9 in., 4 ft. 7 in., 4 ft. 5 in., 4 ft. 3 in., 3 ft. 10 in. Find the average height.

10. The earnings of a company for 1922 were $41,117,-487.28. This was $559,509.99 more than in 1921. What were the earnings for 1921?

11. A company has 257 former employees on its pension roll. The average pension paid is $44 per month. How much is paid per month by this company in pensions? How much per year?

12. A farm of 160 acres was sold for $20,675, including stock valued at $1500 and farming implements at $775. Find the price per acre for the land itself.

13. How many times can a pail holding 3 pints be filled from a jar containing 3 gallons and 3 quarts?

14. How many times can a pail holding 3 pints be filled from a jar containing 7 gallons and 1 pint?

15. John paid $2.00 for 8 dozen oranges. He sold these oranges at the fair at the rate of two for five cents. How much did he make?

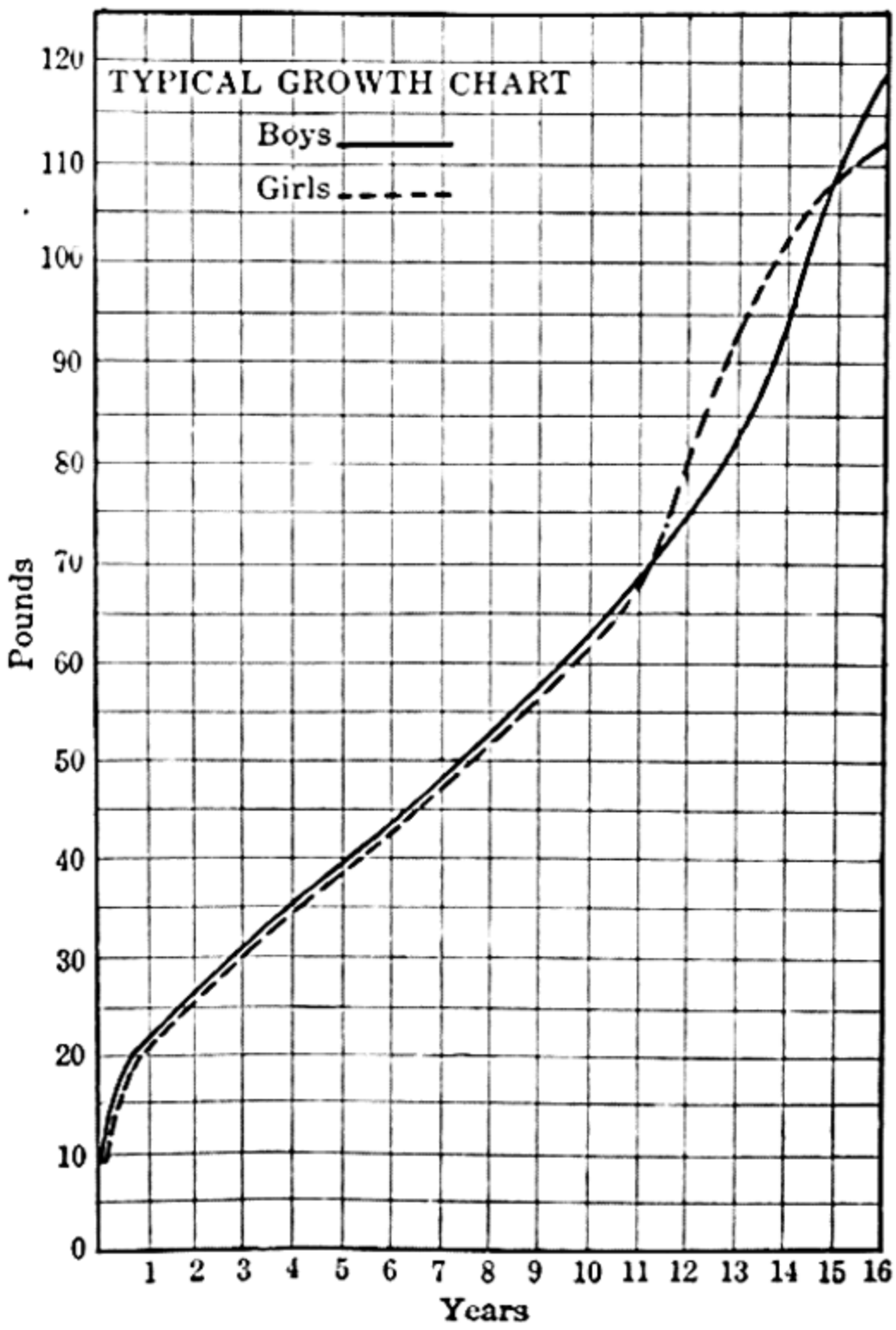


FIG. 7.

16. At 75 cents a square yard, find the cost of 62 square yards of linoleum.

17. The graph in Fig. 7 is like those you have seen so often in the newspapers and you should learn to interpret it. It is a comparison of the average weights of boys and girls up to 16 years of age. What is the average weight of a 16-year-old boy? A 16-year-old girl? A 5-year-old boy? A 3-year-old girl? A 9-year-old boy?

18. Between what ages do the girls weigh, on the average, more than the boys?

19. At what ages (approximately) are their average weights the same?

20. At what age after the first year is the growth most rapid for the boys? For the girls?

21. Let everyone in your room get weighed and compare the average weight of the girls with that of the boys.

22. The graph below taken from a newspaper explains itself. Compare the rate of increase in the number of deaths from automobiles from 1912 to 1921 with the

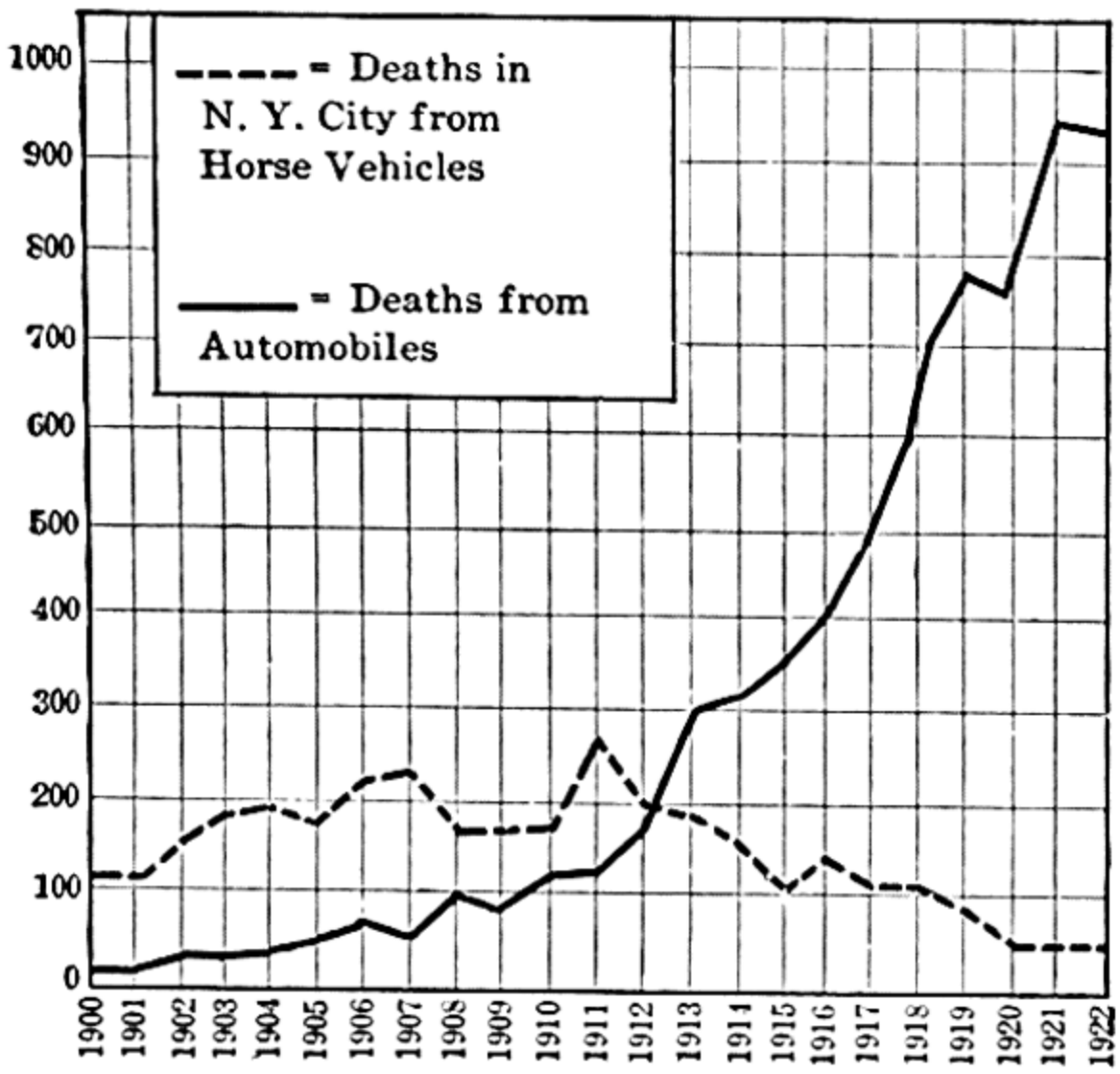


FIG. 8.

rate from 1900 to 1912. What is the nature of the graph when it shows a rapid increase? What do you suppose caused the rapid increase in the number of deaths?

23. Over what period has the deaths from horse-drawn vehicles shown a general decrease? What do you think was the cause?

24. The following table of numbers shows the temperature by hours as recorded by a thermometer. Plot a

graph showing the variation, and explain the causes for the rise and fall of the graph.

6 A.M. 76°	10 A.M. 83°	2 P.M. 93°
7 A.M. 77°	11 A.M. 85°	3 P.M. 92°
8 A.M. 79°	12 M. 89°	4 P.M. 90°
9 A.M. 82°	1 P.M. 92°	5 P.M 80°

25. The Brown family lives in town. There are three children, Sam, Lucy, and the baby. Mr. Brown is a teacher and receives a salary of $225 per month for a term of nine months each year. They own their home, on which there is a mortgage. The following itemized list shows a typical year's expenses:

Interest on mortgage..... $175 *per year*
Taxes and insurance on house..... $ 54 *per year*
Food (groceries, meat, milk, etc.)..... $53 *per month*
Mr. Brown's life insurance policies:
1st policy—annual premium.....$65.58
2nd policy—annual premium..... 70.92
3rd policy—annual premium..... 32.46
For benevolence (church, charity, etc.) one tenth of annual income.
Allowance to Sam to cover all his personal expense..... $12.50 *per month*
Allowance to Lucy for pin money.. 25 *cts. per week*
Expenses for education (books, lectures, etc.)..... $ 75 *per year*
Clothes for Mr. Brown..... $ 85 *per year*
Clothes for Mrs. Brown..... $125 *per year*
Clothes for Lucy and the baby..... $ 75 *per year*

(*a*) What is the total amount received by Mr. Brown as salary for teaching?

(*b*) What is the greatest item of expense for the whole year? How much is it?

(*c*) If $75 is allowed for miscellaneous unexpected expenses, how much can the family save to pay on the mortgage each year?

The family wants to buy an automobile and all agree to help in securing the money.

(*d*) Lucy gets a job as cash girl in a store and earns $5.50 per week for 9 weeks. How much does she earn during the nine weeks?

(*e*) Lucy also agrees to give up her pin money for 12 weeks. What is the total of Lucy's two contributions?

(*f*) Mr. Brown gets a part-time job with a real estate company for 15 weeks during the vacation at $24 per week and does 38 hours tutoring at $1.25 per hour. What is the total amount earned by Mr. Brown for the car?

(*g*) Sam secures the use of some vacant lots and raises a garden. He pays $3.75 for plowing, $6.75 for seed, and $13.43 for tools. He sells:

55 *doz. ears sweet corn* @ 35*c.*
75 *lb. green beans* @ 9*c.*
75 *bunches of radishes* @ 6*c.*
124 *heads of lettuce* @ 8*c.*
58 *lb. spinach* @ 7*c.*
135 *lb. tomatoes* @ 14*c.*
125 *lb. new potatoes* @ 8*c.*
12 *bu. green peas* @ 10*c. per quart.*

What were his total expenses? His total receipts? How much did he have, after deducting expenses, to pay on the auto?

(*h*) What was the total amount earned for the automobile by Lucy, Sam, and Mr. Brown?

QUESTIONS FOR REVIEW

1. Copy the following items, find their sum, and write down the amount of change that you should get from a $20 bill: $3.85; $1.57; $5.17; $2.67; $4.15. (No credit unless the answer is correct.)

2. If each purchase in Problem 1 is made separately and a $10 bill is given in payment for it, state the amount of change that you should receive in each case.

3. What is an addend? A multiplicand? A minuend? A dividend? A multiplier? A divisor? A subtrahend? A sum? A quotient? A product? A remainder?

4. Arrange the following boys in the order in which you think they would be preferred by a business man as helpers. John computes very rapidly and often makes mistakes. He does not take time to check. Charles is very slow and seldom, if ever, makes a mistake. Clarence computes at a medium rate and checks every problem. George works fast and checks every problem quickly, removing all of his few mistakes.

5 Have your records on the tests for proficiency in the fundamental operations steadily improved? If not, why not?

6. Plot on the same sheet the graphs of Mary's and Lucy's records on the same test taken on successive days as given below.

	I	II	III	IV	V	VI	VII	VIII
Mary	8	10	11	12	14	15	16	17
Lucy	8	11	10	12	13	16	14	19

Which is the better record?

7. Why should you practice day after day on a multiplication table? On addition combinations?

CHAPTER III

COMMON FRACTIONS

22. A ***common fraction*** is used to represent the *quotient* of two numbers when the divisor is not contained in the dividend a whole number of times. Thus, 2/3 means the result of dividing (or trying to divide) 2 by 3.

The ***numerator*** represents the dividend and the ***denominator*** the divisor in the indicated division. In the fraction 2/3, 2 is the numerator and 3 the denominator. The numerator and denominator are called the ***terms*** of the fraction.

Sometimes whole numbers are expressed as fractions, as

$$2 = \frac{8}{4}. \qquad 3 = \frac{9}{3} = \frac{30}{10}.$$

If a fraction has the numerator greater than the denominator, the indicated division may be wholly or partly performed. Thus,

$$\frac{125}{25} = 5. \qquad \frac{11}{3} = 3\tfrac{2}{3}.$$

Such a number as $3\frac{2}{3}$, composed of a whole number and a fraction, is called a ***mixed number.***

Some fractional quotients can be expressed easily by means of decimals. Thus,

$$\frac{1}{2} = .5 \qquad \frac{1}{4} = .25$$

The principles of operation for common fractions will be restated as needed in this chapter merely for review.

The numerator and the denominator of a fraction both may be multiplied by or divided by the same number without changing the value of the fraction. By these operations

the fraction is said to be raised to higher or reduced to lower terms. Thus,

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}. \qquad \frac{15}{18} = \frac{15 \div 3}{18 \div 3} = \frac{5}{6}.$$

To reduce a common fraction to a decimal, introduce decimal places in the numerator (by annexing zeros after the decimal point) *and proceed with the indicated division* until there is no remainder or until the number of decimal places is sufficient for the accuracy of the problem.

EXAMPLE 1. *Reduce* 7/25 *to a decimal fraction.*

$$\begin{array}{r} .28 \\ 25\overline{)7.00} \\ \underline{50} \\ 200 \\ \underline{200} \text{ (no remainder)} \end{array}$$

$\frac{7}{25} = .28$

EXAMPLE 2. *Reduce* 11/15 *to a decimal carried out to three decimal places.*

$$\begin{array}{r} .733 \\ 15\overline{)11.000} \\ \underline{105} \\ 50 \\ \underline{45} \\ 50 \\ \underline{45} \\ 5 \text{ (remainder)} \end{array}$$

$\frac{11}{15}$ is approximately equal to .733

Perhaps you always have considered a fraction as a number of parts of a whole. For example, 3/4 means that a whole has been divided into four equal parts, three of which have been taken. This is one correct idea of a fraction. The word "fraction" (derived from the Latin) means that something has been broken up into pieces, and the "part of a whole" idea was probably the prevailing one when fractions were first used.

Long, long ago in the days before historical records were kept, indeed before men had learned to write even with picture symbols, when people could count only by the help of fingers and toes, they had no need for fractions. Their herds could be counted in whole numbers. Their war clubs could be counted in whole numbers. Such an idea as half a tent or half a tree, did not occur to them. As they became more civilized, tilled the fields and raised grain, or in various ways dealt with smaller things, the idea of measuring things as well as counting things gradually entered their minds. Corn and wheat came to be measured by bushels (or by some similar measure) and soon they had half-bushels and other fractional parts. It was thus that the idea of a fraction probably originated. The invention and use of fractions marked a very important step forward in civilization.

Let us imagine a schoolboy of ancient Babylonia in southern Asia as he studied arithmetic more than 6000 years ago. With a hard, blunt-pointed instrument he made peculiar-looking wedge-shaped symbols on a soft, smooth clay surface. He could erase them with a flat board by patting the surface smooth again. His textbook (if he had any) had been written the same way on a sort of clay brick and had been baked hard so that the symbols could not be erased. You wonder whether he had to learn fractions? He did, and to you his fractions may seem clumsy. All his fractions were 60ths, 3600ths, 216000ths, and so on, all the denominators being multiples of 60, just as our decimals are based on 10ths, 100ths, 1000ths, and so on, all the denominators being multiples of 10. He wrote them much as you write decimals, omitting the denominator.

About the same time that the Babylonian youth was dealing with his 60ths, some schoolboy down in Egypt was struggling to express all his fractions with ones for numerators. He did not know such forms as 2/3, 7/8, etc., but used instead $(\frac{1}{2}+\frac{1}{6})$, $(\frac{1}{2}+\frac{1}{4}+\frac{1}{8})$, etc.

A few thousand years later we find a Roman schoolboy basing his fractions on 12ths, or with multiples of 12 as denominators, as the Babylonian used 60ths. We deal with problems in business and in common life that would have been very difficult, or quite impossible, for even the most learned scholars of those ancient times.

In this book we do not discard the idea that a fraction is part of a whole, but we do want you to see also that it is a quotient as defined in the beginning of this section.

23. Addition of Fractions.

To find the sum of two or more fractions having the same denominator, add the numerators and divide this sum by the common denominator. The division may or may not be performed; that is, the sum may be written over the common denominator in the usual way of writing a fraction. Thus,

$$\frac{2}{7}+\frac{3}{7}+\frac{6}{7}=\frac{11}{7}.$$

If the fractions have not the same denominator, they must be raised to higher terms so as to have a common denominator. This is illustrated in the examples below.

EXAMPLE 1. *Find the sum of* $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, *and* $\frac{11}{16}$.

$$\begin{aligned}\tfrac{3}{4} &= \tfrac{36}{48}\\ \tfrac{5}{6} &= \tfrac{40}{48}\\ \tfrac{7}{8} &= \tfrac{42}{48}\\ \tfrac{11}{16} &= \tfrac{33}{48}\\ \hline \tfrac{151}{48} &= 3\tfrac{7}{48}\end{aligned}$$

EXAMPLE 2. *Find the sum of* $17\frac{3}{8}$, $16\frac{5}{16}$, *and* $31\frac{27}{32}$.

$$17\tfrac{3}{8} = 17\tfrac{12}{32}$$
$$16\tfrac{5}{16} = 16\tfrac{10}{32}$$
$$\underline{31\tfrac{27}{32} = 31\tfrac{27}{32}}$$
$$64\tfrac{49}{32} = 65\tfrac{17}{32}$$

To find the *lowest common denominator* of several fractions in the most economical way:

(1) Take the largest denominator (in Example 1: 16).

(2) Try multiples of it (in Example 1: 16, 32, 48, etc.).

(3) Choose the smallest multiple that will contain each of the other denominators (in Example 1: 3 × 16 or 48).

EXERCISES

Carry out each of the following indicated additions:

1. $\frac{1}{2} + \frac{2}{3} + \frac{1}{6}$
2. $\frac{1}{2} + \frac{1}{3} + \frac{5}{6}$
3. $\frac{1}{2} + \frac{2}{5} + \frac{1}{10}$
4. $\frac{1}{2} + \frac{4}{5} + \frac{7}{10}$
5. $\frac{1}{2} + \frac{2}{3} + \frac{1}{12}$
6. $\frac{1}{2} + \frac{2}{3} + \frac{7}{12}$
7. $\frac{2}{3} + \frac{1}{4} + \frac{1}{12}$
8. $\frac{1}{3} + \frac{1}{4} + \frac{7}{12}$
9. $\frac{2}{3} + \frac{1}{4} + \frac{5}{12}$
10. $\frac{2}{3} + \frac{3}{4} + \frac{7}{12}$
11. $\frac{1}{3} + \frac{3}{4} + \frac{11}{12}$
12. $\frac{2}{3} + \frac{3}{4} + \frac{1}{12}$
13. $\frac{2}{3} + \frac{3}{4} + \frac{11}{12}$
14. $\frac{2}{3} + \frac{3}{4} + \frac{5}{12}$
15. $\frac{1}{4} + \frac{1}{5} + \frac{1}{10}$
16. $\frac{1}{4} + \frac{2}{5} + \frac{1}{10}$
17. $\frac{1}{4} + \frac{3}{5} + \frac{1}{10}$
18. $\frac{1}{2} + \frac{2}{3} + \frac{7}{10}$
19. $\frac{1}{4} + \frac{2}{5} + \frac{1}{12}$
20. $\frac{1}{4} + \frac{3}{5} + \frac{1}{12}$
21. $\frac{1}{4} + \frac{2}{5} + \frac{4}{15}$
22. $\frac{1}{4} + \frac{3}{5} + \frac{1}{15}$
23. $\frac{1}{2} + \frac{1}{3} + \frac{1}{15}$
24. $\frac{1}{4} + \frac{1}{5} + \frac{7}{10}$
25. $\frac{1}{4} + \frac{4}{5} + \frac{3}{10}$
26. $\frac{1}{4} + \frac{4}{5} + \frac{7}{10}$
27. $\frac{3}{4} + \frac{5}{6} + \frac{7}{8}$
28. $\frac{3}{4} + \frac{5}{9} + \frac{5}{6}$
29. $\frac{5}{6} + \frac{7}{10} + \frac{3}{4}$
30. $\frac{3}{4} + \frac{5}{6} + \frac{9}{16}$
31. $\frac{7}{8} + \frac{1}{4} + \frac{9}{10}$
32. $\frac{5}{12} + \frac{5}{8} + \frac{3}{4}$
33. $\frac{3}{4} + \frac{5}{8} + \frac{3}{14}$
34. $\frac{3}{10} + \frac{5}{12} + \frac{1}{4}$
35. $\frac{9}{10} + \frac{1}{4} + \frac{15}{16}$
36. $\frac{11}{12} + \frac{3}{4} + \frac{13}{16}$
37. $\frac{5}{6} + \frac{3}{8} + \frac{7}{16}$
38. $\frac{3}{8} + \frac{8}{9} + \frac{5}{12}$
39. $\frac{7}{9} + \frac{3}{10} + \frac{2}{15}$
40. $\frac{3}{4} + \frac{5}{6} + \frac{7}{8} + \frac{5}{16}$
41. $\frac{5}{6} + \frac{3}{8} + \frac{3}{16} + \frac{5}{16}$
42. $\frac{5}{9} + \frac{7}{10} + \frac{11}{12}$
43. $\frac{7}{10} + \frac{3}{4} + \frac{11}{14}$
44. $\frac{1}{2} + \frac{7}{8} + \frac{5}{16} + \frac{7}{32}$
45. $\frac{1}{2} + \frac{7}{8} + \frac{15}{16} + \frac{17}{32}$
46. $\frac{3}{4} + \frac{5}{8} + \frac{13}{16} + \frac{25}{32}$
47. $26\frac{1}{3} + 52\frac{5}{6} + 12\frac{1}{4}$
48. $37\frac{3}{4} + 16\frac{5}{8} + 25\frac{1}{6}$

24. Subtraction of Fractions.

EXAMPLE 1. *Subtract* $9\frac{2}{3}$ *from* $15\frac{3}{4}$.

$$15\tfrac{3}{4} = 15\tfrac{9}{12}$$
$$\underline{9\tfrac{2}{3} = 9\tfrac{8}{12}}$$
$$6\tfrac{1}{12}$$

EXAMPLE 2. *Subtract* $16\frac{3}{4}$ *from* $23\frac{1}{3}$.

$$23\tfrac{1}{3} = 23\tfrac{4}{12} = 22\tfrac{16}{12}$$
$$\underline{16\tfrac{3}{4} = 16\tfrac{9}{12} = 16\tfrac{9}{12}}$$
$$6\tfrac{7}{12}$$

EXERCISES

Carry out each of the following indicated subtractions:

1. From $\frac{15}{16}$ take $\frac{5}{8}$.
2. From $\frac{5}{6}$ take $\frac{3}{4}$.
3. From $\frac{9}{16}$ take $\frac{5}{32}$.
4. From 2 take $\frac{7}{8}$.
5. From 3 take $\frac{3}{4}$.
6. From $5\frac{7}{8}$ take $\frac{5}{8}$.
7. From $7\frac{3}{4}$ take $3\frac{5}{8}$.
8. From $9\frac{5}{6}$ take $2\frac{5}{8}$.
9. From $11\frac{2}{3}$ take $6\frac{3}{4}$.
10. From $12\frac{1}{4}$ take $7\frac{5}{8}$.
11. From $27\frac{3}{8}$ take $17\frac{5}{6}$.
12. From $31\frac{1}{4}$ take $29\frac{7}{8}$.
13. From $45\frac{3}{16}$ take $26\frac{7}{8}$.
14. From $120\frac{5}{8}$ take $97\frac{5}{6}$.
15. From $211\frac{3}{8}$ take $57\frac{3}{10}$.
16. From $312\frac{3}{4}$ take $34\frac{3}{8}$.
17. From $75\frac{9}{16}$ take $29\frac{3}{4}$.
18. From $82\frac{5}{8}$ take $69\frac{2}{5}$.
19. From $211\frac{1}{5}$ take $34\frac{1}{3}$.
20. From $620\frac{1}{8}$ take $240\frac{1}{3}$.
21. From $162\frac{1}{4}$ take $85\frac{1}{3}$.
22. From $75\frac{1}{3}$ take $40\frac{1}{2}$.
23. From $624\frac{1}{5}$ take $62\frac{1}{2}$.
24. From $820\frac{1}{6}$ take $79\frac{1}{2}$.
25. From $1221\frac{3}{8}$ take $48\frac{3}{4}$.
26. From $221\frac{3}{16}$ take $175\frac{3}{8}$.
27. From $422\frac{5}{6}$ take $47\frac{1}{4}$.
28. From $225\frac{2}{3}$ take $120\frac{2}{5}$.
29. From $66\frac{2}{3}$ take $37\frac{1}{2}$.
30. From $83\frac{1}{3}$ take $66\frac{2}{3}$.
31. From $87\frac{1}{2}$ take $16\frac{2}{3}$.
32. From $622\frac{5}{8}$ take $420\frac{5}{6}$.
33. From $200\frac{1}{5}$ take $100\frac{1}{4}$.
34. From $255\frac{1}{6}$ take $202\frac{1}{4}$.
35. From 75 take $47\frac{1}{3}$.
36. From 162 take $95\frac{3}{8}$.
37. From $255\frac{1}{2}$ take $120\frac{5}{6}$.
38. From $111\frac{1}{2}$ take $99\frac{2}{3}$.
39. From $85\frac{1}{2}$ take $40\frac{5}{8}$.
40. From $264\frac{1}{2}$ take $120\frac{7}{8}$.

25. Reducing Mixed Numbers to Entire Fractions.

This is a special case of addition of fractions. The integral part (whole number) of the mixed number is made into a fraction with the same denominator as the fractional part. The numerators of the two fractions are then added.

EXAMPLE. *Reduce* $11\frac{5}{6}$ *to an entire fraction.*

$$11 = \tfrac{66}{6}; \quad \tfrac{66}{6} + \tfrac{5}{6} = \tfrac{71}{6}.$$

This explains the short rule you have probably seen and used; namely,

Multiply the integral part of the mixed number by the denominator of the fractional part and add the numerator of the fraction, writing the sum over the denominator.

It is better to understand the principles underlying a rule than merely to try to memorize and use the rule.

EXERCISES

Reduce to entire fractions. See how many of them you can do mentally.

1. $3\frac{2}{5}$
2. $5\frac{3}{8}$
3. $7\frac{5}{6}$
4. $4\frac{5}{9}$
5. $12\frac{2}{3}$
6. $8\frac{7}{8}$
7. $6\frac{4}{7}$
8. $9\frac{5}{8}$
9. $3\frac{7}{12}$
10. $11\frac{4}{9}$
11. $4\frac{7}{9}$
12. $15\frac{1}{4}$
13. $5\frac{5}{12}$
14. $3\frac{5}{16}$
15. $25\frac{3}{4}$
16. $64\frac{3}{5}$
17. $72\frac{4}{9}$
18. $36\frac{1}{2}$
19. $73\frac{1}{8}$
20. $46\frac{3}{4}$
21. $76\frac{1}{6}$
22. $65\frac{2}{3}$
23. $47\frac{5}{8}$
24. $95\frac{5}{7}$
25. $203\frac{1}{2}$
26. $54\frac{1}{4}$
27. $101\frac{5}{8}$
28. $66\frac{2}{3}$
29. $37\frac{1}{2}$
30. $62\frac{1}{2}$
31. $47\frac{2}{3}$
32. $17\frac{5}{7}$
33. $23\frac{5}{6}$
34. $24\frac{4}{5}$
35. $103\frac{1}{3}$
36. $14\frac{5}{16}$

26. Multiplication of Fractions.

You are somewhat familiar with the principles of multiplication and division of common fractions. If you consider a fraction either as a number of parts of a whole or as the quotient of a division, it is easy to develop these principles. The most important of all the ideas connected with multiplication will be stated again.

What is two times three apples? Two times three books? Two times three fifths?

2×3 apples $= 6$ apples. 2×3 books $= 6$ books.

$$2 \times 3/5 = 6/5.$$

A fraction may be multiplied by any number by multiplying the numerator by the given number.

CAUTION. Do *not* multiply *both* terms of a fraction by the same number if you want to multiply the fraction. That would merely raise the fraction to higher terms without changing its value. (See page 51.)

$$2 \times \tfrac{8}{9} = \tfrac{16}{9} \text{ [}\textit{not } \tfrac{16}{18}\text{]}.$$

A fraction may be divided by any number by dividing the numerator by the given number.

CAUTION. Do *not* divide *both* terms of a fraction by the same number if you want to divide the fraction.

The two principles stated above are fundamental and must be kept always in mind.[1]

The following examples furnish a review of the familiar methods of multiplication.

EXAMPLE 1. *Multiply* $\frac{8}{9}$ *by* 1232.

$$1232 \times \tfrac{8}{9} = \tfrac{9856}{9} = 1095\tfrac{1}{9}.$$

[1] There is another way to multiply a fraction, that is, by dividing its denominator. Also there is another way to divide a fraction, that is, by multiplying its denominator.

EXAMPLE 2. *Find* $\frac{7}{8}$ *of* 6928.

$$\frac{7}{8} \text{ of } \overset{866}{6928} = 6062.$$

EXAMPLE 3. *Multiply* $\frac{9}{16}$ *by* $\frac{2}{5}$.

(*a*) $\frac{2}{5} \times \frac{9}{16} = \frac{18}{80}$; $\frac{18}{80} = \frac{9}{40}$. (*b*) $\frac{2}{5} \times \frac{9}{\underset{8}{16}} = \frac{9}{40}$.

EXAMPLE 4. *Multiply* $56\frac{2}{3}$ *by* 8.

$$\begin{array}{r} 56\frac{2}{3} \\ 8 \\ \hline 448 \\ 5\frac{1}{3} \\ \hline 453\frac{1}{3} \end{array}$$

EXAMPLE 5. *Multiply* $12\frac{2}{3}$ *by* $3\frac{1}{2}$.

METHOD (*a*)

$$\begin{array}{r} 12\frac{2}{3} \\ 3\frac{1}{2} \\ \hline 36 \\ 2\ (3 \times \frac{2}{3}) \\ 6\ (\frac{1}{2} \text{ of } 12) \\ \frac{1}{3}\ (\frac{1}{2} \text{ of } \frac{2}{3}) \\ \hline 44\frac{1}{3} \end{array}$$

METHOD (*b*)

$$12\tfrac{2}{3} = \tfrac{38}{3}$$
$$3\tfrac{1}{2} = \tfrac{7}{2}$$
$$\tfrac{\overset{19}{38}}{3} \times \tfrac{7}{2} = \tfrac{133}{3}$$
$$= 44\tfrac{1}{3}$$

EXERCISES

A. Carry out each of the following multiplications:

1. $564 \times \frac{7}{8}$
2. $348 \times \frac{5}{6}$
3. $756 \times \frac{7}{12}$
4. $313 \times \frac{3}{4}$
5. $1232 \times \frac{8}{9}$
6. $26\frac{3}{4} \times 9$
7. $37\frac{2}{3} \times 8$
8. $76\frac{3}{8} \times 12$
9. $84\frac{7}{9} \times 15$
10. $\frac{5}{6}$ of $\frac{7}{12}$
11. $\frac{8}{9}$ of $\frac{11}{12}$
12. $\frac{9}{10}$ of $\frac{15}{16}$

B. In the following exercises both multiplicand and multiplier are mixed numbers. Find the product in each case.

1. $8\frac{2}{3} \times 6\frac{3}{4}$
2. $12\frac{3}{5} \times 20\frac{3}{8}$
3. $24\frac{5}{6} \times 3\frac{3}{4}$
4. $61\frac{1}{8} \times 9\frac{1}{2}$
5. $29\frac{3}{8} \times 8\frac{1}{3}$
6. $564\frac{3}{4} \times 7\frac{1}{5}$
7. $875\frac{2}{3} \times 8\frac{2}{3}$
8. $568\frac{3}{4} \times 5\frac{1}{2}$
9. $579\frac{5}{8} \times 6\frac{3}{4}$

27. Aliquot Parts of 100. An *aliquot part* of a number is a number that is contained in it an integral number of times.

For example, 50, 25, $33\frac{1}{3}$, 20, $16\frac{2}{3}$, and 10 are aliquot parts of 100. These aliquot parts respectively are: 1/2, 1/4, 1/3, 1/5, 1/6, and 1/10 of 100.

28. Multiplication by Aliquot Parts of 100. To multiply a number by 100, you annex two zeros if it is a whole number; if it is a decimal, you move the decimal point two places to the right. For example, $100 \times 24 = 2400$; $100 \times 3.24 = 324$; $100 \times 0.34 = 34$; etc.

To multiply a number by an aliquot part of 100, you first multiply it by 100; then you take the fractional part part of this product. For example,

$$25 \times 36 = \frac{100}{4} \times 36 = 900; \quad 33\tfrac{1}{3} \times 42 = \frac{100}{3} \times 42 = 1400.$$

NOTE. When the division will result in an integer, it is easier to divide first and then multiply. For example, $25 \times 36 = 1/4$ of $36 \times 100 = 900$.

EXERCISES

1. 50×44
2. 50×56
3. 50×78
4. 50×92
5. 50×23
6. 50×6.4
7. 50×13.6
8. 50×35.4
9. 50×8.6
10. 50×1.35
11. 25×36
12. 25×48
13. 25×60
14. 25×84
15. 25×96
16. 25×18
17. 25×22
18. 25×25
19. 25×3.4
20. 25×9.4
21. $33\frac{1}{3} \times 24$
22. $33\frac{1}{3} \times 54$
23. $33\frac{1}{3} \times 63$
24. $33\frac{1}{3} \times 72$
25. $33\frac{1}{3} \times 84$
26. $33\frac{1}{3} \times 96$
27. $33\frac{1}{3} \times 87$
28. $33\frac{1}{3} \times 90$
29. $33\frac{1}{3} \times 9.9$
30. $33\frac{1}{3} \times 8.1$
31. $16\frac{2}{3} \times 18$
32. $16\frac{2}{3} \times 36$
33. $16\frac{2}{3} \times 42$

34. $16\frac{2}{3} \times 60$
35. $16\frac{2}{3} \times 72$
36. $16\frac{2}{3} \times 84$
37. $16\frac{2}{3} \times 90$
38. $16\frac{2}{3} \times 54$
39. $16\frac{2}{3} \times 5.7$

29. Multiplication by Aliquot Parts of $1.00.

When the multiplicand and multiplier are both abstract numbers, you may interchange them; when one of them is concrete, you may disregard this fact until you desire to label your answer. By doing this, you may often save labor in multiplying when you are finding the cost of articles purchased at a fractional part of a dollar for each article. For example, $96 \times 33\frac{1}{3}¢ = \frac{1}{3}$ of $96 \times \$1 = \frac{1}{3}$ of $\$96 = \$32$; $144 \times 16\frac{2}{3}¢ = \frac{1}{6} \times \$144 = \$24$.

EXERCISES

In each of the following exercises find the cost of each article and the total cost:

1. 14 lb. at 50¢ per lb.
   23 lb. at 25¢ per lb.
   33 lb. at 50¢ per lb.
2. 35 lb. at 50¢ per lb.
   45 lb. at 20¢ per lb.
   26 lb. at 25¢ per lb.
3. 16 yd. at 10¢ per yd.
   18 yd. at 25¢ per yd.
   25 yd. at 50¢ per yd.
4. 15 doz. at 25¢ per doz.
   19 doz. at 50¢ per doz.
   25 doz. at 20¢ per doz.
5. 18 qt. at 50¢ per qt.
   32 qt. at 25¢ per qt.
   26 qt. at 10¢ per qt.
6. 60 doz. at 25¢ per doz.
   40 doz. at 50¢ per doz.
   80 doz. at 25¢ per doz.
7. 42 lb. at $33\frac{1}{3}¢$ per lb.
   48 lb. at $16\frac{2}{3}¢$ per lb.
   78 lb. at 25¢ per lb.
8. 124 doz. at 50¢ per doz.
   63 doz. at $33\frac{1}{3}¢$ per doz.
   72 doz. at $16\frac{2}{3}¢$ per doz.
9. 76 qt. at 25¢ per qt.
   84 qt. at $33\frac{1}{3}¢$ per qt.
   96 qt. at $16\frac{2}{3}¢$ per qt.
10. 17 doz. at 50¢ per doz.
    52 doz. at 25¢ per doz.
    35 doz. at 20¢ per doz.

30. Division of Fractions.

EXAMPLE 1. *Divide* $\frac{17}{18}$ *by* 3.

$$\frac{17}{18} \div 3 = \frac{17}{18} \times \frac{1}{3} = \frac{17}{54}.$$

EXAMPLE 2. *Divide* $\frac{15}{16}$ *by* $\frac{3}{4}$.

$$\frac{15}{16} \div \frac{3}{4} = \frac{\overset{5}{\cancel{15}}}{\underset{4}{\cancel{16}}} \times \frac{\cancel{4}}{\cancel{3}} = \frac{5}{4} = 1\tfrac{1}{4}.$$

EXAMPLE 3. *Divide* $5\frac{1}{3}$ *by* $3\frac{1}{2}$.

$$5\tfrac{1}{3} \div 3\tfrac{1}{2} = \tfrac{16}{3} \div \tfrac{7}{2} = \tfrac{16}{3} \times \tfrac{2}{7} = \tfrac{32}{21} = 1\tfrac{11}{21}.$$

EXAMPLE 4. *Divide* $15\frac{3}{5}$ *by* $3\frac{2}{3}$.

$$15\tfrac{3}{5} \div 3\tfrac{2}{3} = \tfrac{78}{5} \div \tfrac{11}{3} = \tfrac{78}{5} \times \tfrac{3}{11} = \tfrac{234}{55} = 4\tfrac{14}{55}.$$

EXERCISES

Carry out each of the folowing indicated divisions:

1. $\frac{8}{9} \div 2$
2. $\frac{3}{8} \div 2$
3. $\frac{7}{8} \div 2$
4. $\frac{7}{8} \div 4$
5. $\frac{3}{4} \div 4$
6. $3 \div \frac{7}{8}$
7. $8 \div \frac{3}{4}$
8. $9 \div \frac{2}{3}$
9. $12 \div \frac{5}{6}$
10. $\frac{7}{8} \div \frac{1}{4}$
11. $\frac{15}{16} \div \frac{3}{4}$
12. $\frac{9}{12} \div \frac{3}{4}$
13. $\frac{3}{5} \div \frac{3}{7}$
14. $\frac{17}{24} \div \frac{8}{9}$
15. $\frac{19}{32} \div \frac{5}{8}$
16. $5\frac{1}{3} \div 3$
17. $16\frac{3}{4} \div 4$
18. $18\frac{2}{3} \div 6$
19. $21\frac{5}{6} \div 6$
20. $24\frac{7}{8} \div 8$
21. $51\frac{1}{3} \div 7$
22. $23\frac{1}{2} \div 11$
23. $12\frac{2}{3} \div 5$
24. $41\frac{2}{5} \div 6$
25. $127\frac{1}{4} \div 12$
26. $426\frac{2}{3} \div 15$
27. $201\frac{1}{2} \div 13$
28. $512\frac{3}{5} \div 21$
29. $6\frac{2}{3} \div 2\frac{1}{2}$
30. $13\frac{1}{2} \div 3\frac{3}{4}$
31. $16\frac{7}{8} \div 2\frac{1}{2}$
32. $25\frac{4}{5} \div 3\frac{1}{3}$
33. $29\frac{1}{3} \div 5\frac{2}{3}$
34. $51\frac{2}{3} \div 4\frac{1}{6}$
35. $125\frac{1}{2} \div 7\frac{1}{4}$
36. $241\frac{2}{5} \div 6\frac{1}{10}$
37. $712\frac{1}{3} \div 51\frac{1}{2}$
38. $69\frac{4}{10} \div 5\frac{3}{10}$
39. $121\frac{7}{10} \div 7\frac{1}{10}$
40. $235\frac{9}{10} \div 3\frac{7}{10}$
41. $5\frac{17}{100} \div 3\frac{41}{100}$
42. $15\frac{23}{100} \div 11\frac{51}{100}$

31. Sums of Common and Decimal Fractions. To find the sum of a common fraction and a decimal fraction, either reduce both of them to common fractions or both to decimal fractions.

EXAMPLE 1. *Add* $\frac{2}{3}$ *and* .5

$$\tfrac{2}{3} + .5 = \tfrac{2}{3} + \tfrac{1}{2} = \tfrac{2}{6} + \tfrac{3}{6} = \tfrac{5}{6}.$$

EXAMPLE 2. *Add* $2\frac{1}{3}$ *and* 7.2

$$2\tfrac{1}{3} + 7.2 = 2\tfrac{1}{3} + 7\tfrac{2}{10} = 2\tfrac{1}{3} + 7\tfrac{1}{5} = 9\tfrac{8}{15}.$$

EXAMPLE 3. *Add* $5\frac{3}{4}$ *and* 3.46

$$5\tfrac{3}{4} + 3.46 = 5.75 + 3.46 = 9.21$$

EXERCISES

Add together all of the numbers in each of the following exercises:

1. $12\frac{1}{2}$, 13.2, 17.8, $16\frac{1}{5}$, $8\frac{4}{5}$
2. $85\frac{1}{3}$, $78\frac{1}{6}$, 38.25, 96.5, $87\frac{1}{3}$
3. $69\frac{3}{8}$, 75.5, 64.5, $68\frac{3}{8}$, $56\frac{7}{8}$
4. 29.75, $69\frac{7}{8}$, $79\frac{3}{8}$, $46\frac{1}{8}$, 28.25
5. $83\frac{5}{6}$, $47\frac{2}{3}$, 87.25, 67.125, $81\frac{1}{3}$
6. 46.3, 78.27, $67\frac{4}{5}$, 78.2, $98\frac{3}{5}$
7. $86\frac{2}{3}$, $58\frac{5}{6}$, 92.8, 87.9, $69\frac{1}{3}$
8. $95\frac{4}{5}$, 67.8, 89.3, $76\frac{1}{4}$, $56\frac{3}{8}$
9. $77\frac{7}{10}$, 66.6, $88\frac{4}{5}$, $55\frac{1}{2}$, 99.9
10. 316.3, $21\frac{2}{5}$, $817\frac{5}{6}$, $86\frac{3}{4}$, 718.7
11. $207\frac{3}{8}$, $73\frac{3}{4}$, 37.25, 7.125, 812.5
12. 3.75, $34\frac{5}{8}$, 86.25, $57\frac{7}{8}$, 94.8
13. 8.375, $33\frac{3}{8}$, $88\frac{7}{8}$, 87.60, $62\frac{5}{8}$
14. 756.12, 34.625, 8.875, 437.8, 8.67

Exercises will be found in Chapter VIII for drill in the operations on fractions and to test your proficiency.

MISCELLANEOUS PROBLEMS

1. A carload of 850 bushels of corn costs $637.50. What is the cost per bushel?

2. A company paid $111,384 to 138 dependents of its employees. What was the average amount paid to each dependent?

3. At $42 a dozen what will $8\frac{3}{4}$ dozen hats cost?

4. The population of the State of New York in 1920 was 10,885,277. Its area is 49,204 square miles. Find the number of people per square mile in the State of New York.

5. A man pays $7215 for a farm of 78 acres. What does he pay per acre?

6. A cubic foot of water weighs 62.5 pounds. What is the weight of 264 cubic feet of water?

7. How many cubic feet in 215,042 cubic inches?

8. An acre of land costs $225. Find the cost of $\frac{5}{8}$ of an acre.

9. A lot of land containing 5250 square feet is purchased at 18 cents a square foot. Find the cost.

10. A bolt of cloth containing 55 yards is cut into overcoats. Allowing $3\frac{1}{4}$ yards for a coat, how many coats can be cut? How many yards will be wasted?

11. When velvet is selling at $4.50 a yard, how much should be paid for $12\frac{3}{8}$ yards?

12. Find the cost of 225 tons of coal at $12.75 per ton.

13. A farmer raised 1200 bushels of potatoes. He sold $\frac{3}{8}$ of them at $1.50 per bushel; the remainder at $1.25 per bushel. How much did he receive for his crop of potatoes?

14. A truck gardener raised 15,000 bunches of celery. He sold $\frac{3}{5}$ of them at 12 cents a bunch; the remainder

at 10 cents a bunch. How much did he receive for his crop of celery?

15. In a certain school of 54 pupils $\frac{5}{8}$ are girls. How many boys are in this school?

16. It takes $\frac{7}{8}$ of a yard of braid for a coat; how many yards are needed for 12 coats? At 27 cents per yard what will the braid for the 12 coats cost?

17. If $\frac{5}{8}$ of a yard of silk costs $2.00, what is the price of a yard? What will $\frac{1}{8}$ of a yard cost?

18. For $\frac{7}{8}$ of a yard of velvet $4.20 is paid. What price is this per yard?

19. For $1\frac{1}{4}$ yards of velvet $6.00 is paid. What price is this per yard? What will $\frac{1}{4}$ of a yard cost?

20. Find the total number of yards in the following pieces of cloth: $25\frac{3}{4}$ yd., $78\frac{3}{4}$ yd., $56\frac{5}{8}$ yd., $46\frac{3}{8}$ yd., $65\frac{7}{8}$ yd., $38\frac{3}{8}$ yd.

21. At $\$24\frac{3}{4}$ each, find the cost of 16 tables.

22. At $\$26\frac{1}{4}$ each, find the cost of 24 tables.

23. A distance of 125 miles is covered by automobile in $4\frac{1}{2}$ hours. What is the average distance traveled per hour?

24. A turkey weighing $16\frac{3}{8}$ pounds is bought at 44 cents a pound. How much is paid for this turkey?

25. A man pays $426 for the rent of an apartment for one year? What does he pay a month?

26. A man pays $4 a week for the rent of a house. What does he pay a month? (52 weeks in a year.)

27. A roast of beef weighing $8\frac{3}{4}$ pounds is bought at 33 cents a pound. What is the cost?

28. When eggs are retailing at 54 cents per dozen, how much should you pay for 32 eggs?

29. A train averages 28 miles per hour. How far will it go in $12\frac{2}{5}$ hours?

30. A merchant paid \$8.75 per barrel for 562 barrels of flour. How much did the flour cost him?

31. A farmer receives the following memoranda from the merchant to whom he has just delivered produce: 14 doz. and 4 eggs at 22 cents per doz.; $7\frac{3}{4}$ lb. butter at 27 cents per lb.; 3 bu. and 30 lb. apples (one bushel = 50 lb.) at \$1.25 per bu.; $30\frac{1}{2}$ lb. tomatoes at 9 cents per lb. How much was due the farmer?

32. A carton containing 75 apples costs \$3.00. What is the average cost of an apple? Before the grocer disposes of all these apples, one fifth of them are lost by decay. The apples are sold at 5 cents each. How much is gained?

33. A carton containing 50 apples costs \$1.50. What is the cost of an apple? If one fifth of these decay and the remainder are sold at 5 cents each, how much is profit?

34. At a ball park there are 40,000 people; four fifths travel by trolley cars. Estimating that there are 75 persons in each car, how many cars are needed to carry this crowd? At 5 cents each what is the total amount of fares collected?

35. If the rest of the crowd at the ball park travel by automobile and each automobile averages 4 people, how many automobiles are needed? At 25 cents each for parking these automobiles, how much is received?

36. The seventh grade class of a big city school went to the country one Saturday for a picnic. They met at the school building and rode in street cars to the railroad station where they boarded a suburban accommodation train. They returned in three hired automobiles. The

party included 33 pupils and the teacher. They agreed to share the expenses of the picnic equally.

(*a*) The street car fare was seven cents each. What was the total amount of this item of expense?

(*b*) The railroad fare each way was 28 cents for each person. The railroad company sold 25-trip tickets for \$3.25 each and 10-trip tickets for \$1.85 each. Would it cost them less to buy one 25-trip ticket and 9 single tickets, or to buy a 25-trip ticket and a 10-trip ticket, and throw away the remaining ticket? Compute the cost of railroad fares by each plan before answering this question.

(*c*) The committee on provisions bought the following supplies: enough buns, doughnuts, and bananas to provide each person three buns, two doughnuts, and one banana; the buns cost 10 cents per dozen, the doughnuts 25 cents per dozen, and the bananas 30 cents per dozen; $4\frac{1}{2}$ lb. of sandwich meat cost 26 cents per lb.; $2\frac{1}{2}$ lb. of pickles, 8 cents per lb.; three watermelons, weighing 21, 27, and 25 lb. respectively, $2\frac{1}{2}$ cents per lb. What was the total amount spent for food?

(*d*) Each of the three automobiles that brought them home charged \$3.00 for the trip. How much was this item of expense?

(*e*) Find the average cost of the picnic for each person who went, assuming that the railroad tickets were bought by the cheapest plan.

(*f*) Suppose that the pupils and teacher agree that each shall pay 75 cents, and that the extra money shall be used to purchase supplies for poor children in the hospital. How much money will they have for this purpose?

QUESTIONS FOR REVIEW

1. What is a common fraction? A decimal fraction?

2. What are the terms of a fraction? Which one represents a divisor? What does the other represent?

3. What kind of denominators did the Babylonians use? The Romans?

4. What kind of numerators did the Egyptians use?

5. What is a mixed number? Give an example.

6. Find the sum of $\frac{5}{8}$, $\frac{7}{12}$, $\frac{9}{16}$, and $\frac{29}{32}$.

7. Find the sum of $\frac{2}{3}$, $\frac{7}{9}$, $\frac{11}{12}$, and $\frac{111}{114}$.

8. From $1001\frac{3}{8}$ subtract $301\frac{5}{12}$.

9. To multiply $\frac{5}{6}$ by 7 do you multiply the numerator or the denominator or both terms by 7?

10. Find the product: $8\frac{5}{12} \times 12\frac{5}{8}$.

11. Find the quotient: $8\frac{5}{12} \div 12\frac{5}{8}$.

12. Tell how you get the lowest common denominator for $\frac{1}{3}$ and $\frac{2}{5}$. Do you use the same method for $\frac{1}{3}$ and $\frac{5}{6}$? Why not?

13. Find the sum of $3\frac{2}{5}$, 4.25, $7\frac{3}{8}$, 9.6, $2\frac{3}{4}$, $8\frac{5}{8}$, and $5\frac{7}{8}$.

14. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{16}$ to decimals carried out to four places. How many of them are exact? Do you notice any numerical relation between the denominators of the fractions for which you found it necessary to approximate the fourth decimal place?

CHAPTER IV

EQUATIONS—RATIOS—PERCENTAGE

I. Equations

32. Equations. The following exercises, of which several are solved, will indicate sufficiently the methods to be used in solving simple equations.

EXERCISES

1. If an orange costs 4 cents, what will 5 oranges cost?

Ans. 5×4 cents, or 20 cents.

2. If an orange costs n cents, what will 5 oranges cost?

Ans. 5 n cents.

(The sign × between 5 and n is omitted.)

3. If one baseball weighs 8 ounces, what will 6 baseballs weigh?

4. If one baseball weighs b ounces, what will 6 baseballs weigh?

5. If a boy is paid 30 cents a day for carrying bundles, how much is he paid for 10 days' work? For $4\frac{1}{2}$ days' work?

6. If a boy is paid n cents a day for carrying bundles, how much is he paid for 10 days' work? For $4\frac{1}{2}$ days' work?

7. If tea is being sold at 60 cents a pound, how much will 3 lb. cost? $\frac{1}{2}$ lb.?

8. If tea is being sold at t cents a pound, how much will 3 lb. cost? $\frac{1}{2}$ lb.?

9. A train travels at a rate of 35 miles an hour. What does 3×35 represent? What does $4\frac{1}{2} \times 35$ represent?

10. A train travels at a rate of m miles an hour. What does $3\,m$ represent? What does $4\frac{1}{2}\,m$ represent?

In Exercises 1–10 you have seen how we use a letter to stand for some number. You learned in the historical story in the first chapter that people have in the past used letters for numbers. The Greeks and Romans had each letter stand for only one number. As we intend now to use letters for numbers, the same letter may stand for several different numbers but for only *one* number at a time. The n of Exercise 2 and the n of Exercise 6 would not be likely to be the same number.

If you are to use this new kind of numbers successfully, you must know how to add them.

11. How many are 4 books and 5 books?

Ans. 9 books.

12. How many are $4\,b$ and $5\,b$?

Ans. $9\,b$.

13. Find the sum of 7 bushels and one bushel.

14. Find the sum of $7\,b$ and b. [NOTE. b means one b. The 1 is not written.]

15. Find the sum of 3 doz., 7 doz., and $\frac{3}{4}$ doz.

16. Find the sum of $3\,d$, $7\,d$, and $\frac{3}{4}\,d$.

17. $3n + 8n + 5n + 7n + 14n + 23n = ?$

18. $4a + a + \frac{3}{4}a + 5a + 2a = ?$

19. Six National League baseballs weigh 54 ounces. What is the weight of one baseball?

SOLUTION. Six times the weight of one baseball = 54 ounces. If you let w stand for the number of ounces in the weight of one baseball, you get $6\,w = 54$. The statement $6\,w = 54$ is an ***equation***. To solve the equation $6\,w = 54$ means to find the value of w that multiplied by 6 gives 54. That value is 9, for 6 times 9 = 54.

Ans. The weight of one baseball = 9 ounces.

The two members (or sides) of an equation are like a pair of scales balanced. If you change the weight in one scalepan, you must change the weight in the other scalepan by just the same amount if the scales are to stay balanced. If you divide one member (or side) of an equation by 3, you must also divide the other member by 3.

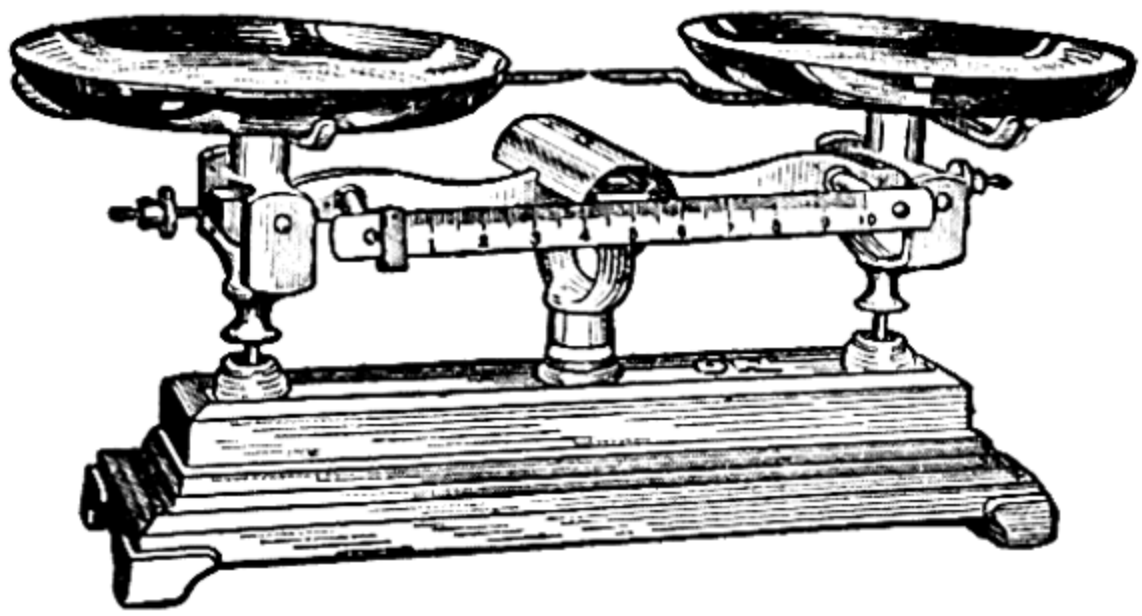


FIG. 9.

The kind of weights in one scalepan can be changed without affecting the balance if the total weight remains the same. A 10-lb. weight can replace a 4-lb. and a 6-lb. weight. In an equation $10a$ may replace $4a$ and $6a$.

20. A boy goes 18 miles on his bicycle in 3 hours; how many miles does he go in an hour?

SOLUTION. If you let m stand for the number of miles that he goes in an hour, the equation is

$$3\ m = 18 \text{ mi.}$$

To get m you divide $3\ m$ by 3, and also divide 18 by 3:

$$m = 6 \text{ mi.}$$

Ans. The boy goes 6 miles per hour.

The solution of $3\ m = 18$ may be arranged as follows:

$$3\ m = 18.$$

Divide each member by 3: $m = 6$.

21. A man gets a weekly salary of \$18. If he works six days, how much does he get per day?

SOLUTION. If d stands for the number of dollars he gets per day, the equation is

$$6\,d = \$18.$$

Divide each member by 6:

$$d = \$3.$$

Each problem should be checked; that is, the number value of the letter should be made to satisfy the conditions of the problem.

CHECK. In this example the check is $6 \times 3 = 18$.

Ans. The man gets \$3 per day.

22. One dozen Reach baseballs cost \$15. What is the price of one ball?

SOLUTION. Let p = the number of dollars that one ball costs; then

$$12\,p = \$15.$$

Divide each member by 12:

$$p = \$1\tfrac{1}{4}.$$

CHECK. $12 \times 1\frac{1}{4} = 15$.

Ans. The price of one baseball = \$1.25.

23. A battleship covering 675 miles in 30 hours travels how many miles per hour?

SOLUTION Let m = number of miles per hour; then

$$30\,m = 675 \text{ mi.}$$

Divide each member by 30:

$$m = 22\tfrac{1}{2} \text{ mi.}$$

CHECK. $30 \times 22\frac{1}{2} = 675$.

Ans. The battleship travels $22\frac{1}{2}$ miles per hour.

(This means that $22\frac{1}{2}$ miles is the average rate per hour of the battleship. Its rate for the first hour might be less than $22\frac{1}{2}$ miles, for the second more than $22\frac{1}{2}$ miles.)

The equation can be used to solve problems far more difficult than the last five. See whether you can solve the following problem without using an equation.

24. Three brothers, James, Henry, and Sam, agreed to pick a farmer's peaches for $7.50, the money to be divided according to how many peaches each boy picked. James picked twice as many bushels as Henry, and Sam three times as many as Henry. How much money did each get?

If we make use of letters for unknown numbers and use the equation, we have the following solution.

SOLUTION. Let n = the number of dollars Henry received.

Then $2\ n$ = the number of dollars James received, for he picked twice as many bushels as Henry.

$3\ n$ = the number of dollars Sam received.

Since the sum of the number of dollars they received was 7.50,

$$n + 2\ n + 3\ n = 7.50$$

If now we replace the left member by the sum of the n's, we have

$$6\ n = 7.50$$

Divide both sides of the equation by 6:

$$n = 1.25$$

Hence $2\ n = 2.50$,

and $3\ n = 3.75$

CHECK. We note that James's share is twice Henry's, and Sam's is three times Henry's. If we add their three shares, we get, $\$1.25 + \$2.50 + \$3.75 = \$7.50$. These values fit the requirements of the problem.

Ans. Henry, $1.25, James, $2.50, Sam, $3.75.

25. Solve the equation $5a = 15$.

SOLUTION. Divide each number by 5:

$$a = 3.$$

CHECK. $5 \times 3 = 15$. *Ans.* $a = 3$.

26. Solve the equation $14 = 4\,y$.

SOLUTION. Divide each member by 4:

$$3\tfrac{1}{2} = y.$$

CHECK. $14 = 4 \times 3\tfrac{1}{2}$. *Ans.* $y = 3\tfrac{1}{2}$.

NOTE. The value of the unknown quantity found by solving an equation is often called the ***root*** of that equation. Thus $3\tfrac{1}{2}$ is called the ***root*** of the equation $14 = 4\,y$.

27. Solve the equation $4 = 28\,m$.

SOLUTION. Divide each member by 28:

$$\frac{1}{7} = m.$$

CHECK. $4 = 28 \times \frac{1}{7}$. *Ans.* $m = \frac{1}{7}$.

What is the root of the equation $4 = 28\,m$?

28. Solve the equation $6.8\,p = 3.4$.

SOLUTION. Divide each member by 6.8:

$$p = \frac{3.4}{6.8} = \frac{1}{2}.$$

CHECK. $6.8 \times \frac{1}{2} = 3.4$ *Ans.* $p = \frac{1}{2}$.

NOTE. In solving Exercises 19–28, we have used the following axiom.*

AXIOM. *If equal numbers are divided by the same number, the quotients are equal.*

* An axiom is the statement of an accepted truth.

Solve and check the following equations:

29. $5\,a = 35.$
30. $2\,x = 28.$
31. $12 = 3\,y.$
32. $28 = 4\,m.$
33. $39 = 13\,h.$
34. $5\,m = 17.$
35. $20 = 5\,b.$
36. $5\,r = 47.$
37. $8\,a = 2.$
38. $6 = 24\,b.$
39. $27\,q = 9.$
40. $5.3\,x = 1.59$
41. $18\,k = 5.4$
42. $12.4\,c = 49.6$
43. $18.4\,r = 280.6$
44. $.65 = .13\,b.$
45. $.035 = .7\,t.$
46. $98 = 1.4\,n.$
47. $54\,r = 65.61$
48. $3\frac{1}{2}\,h = 7.$
49. $2\frac{1}{2}\,a = 10.$
50. $3\frac{1}{2}\,k = 20.$
51. $12 = 2\frac{2}{5}\,x.$
52. $18 = 4\frac{1}{2}\,z.$
53. $4\frac{2}{3}\,x = 42.$
54. $7\frac{1}{4}\,a = 32\frac{5}{8}.$
55. $16\frac{1}{2}\,b = 49\frac{1}{2}.$
56. $25 = 3\frac{1}{8}\,m.$
57. $9\frac{1}{8}\,x = 146.$
58. $\frac{1}{5}\,x = 7.$
59. $\frac{1}{2}\,k = 7\frac{1}{2}.$
60. $19 = \frac{1}{3}\,x.$
61. $24 = \frac{2}{3}\,y.$
62. $\frac{3}{4}\,a = 12.$
63. $\frac{3}{8}\,b = 15.$
64. $35 = \frac{5}{16}\,c$
65. $3\,a + 6\,a = 36.$
66. $5\,x + 7\,x + x = 52.$
67. $2\,c + 8\,c + 4\,c = 35.$
68. $b + 2b + 3b = 20.$
69. $y + 11\,y + 20\,y = 6.4$
70. $.4\,x + 1.6\,x + 5\,x = 28.$
71. $n + .5\,n + 3\,n = 2.7$
72. $17\,m + 26\,m + 38\,m = 1701.$

By the same method as in Exercises 19–24 form equations for each of the following problems, and solve the equations. Check each answer.

73. A man earns $700 on a job requiring 28 weeks. How much are his earnings per week?

74. An automobile goes 165 miles in 6 hours. What is the average rate in miles per hour?

75. If you should save 75 cents a week, in how many weeks would you have enough money to buy a bicycle valued at $19.50?

76. A freight car was built to carry a load of 34,720 lb. How many tons of coal could it carry? (A ton of coal weighs 2240 pounds, called a *long ton*.)

77. An aviator traveled 633 miles in 6 hours. What was his average rate in miles per hour?

78. An express train goes from Boston, Mass., to Portland, Me., a distance of 108 miles, in three hours. What is its average rate per hour?

79. A jitney-bus operator expects to take in not less than $6.50 per day at 5 cents a passenger. How many passengers, at least, must he have?

80. There are 2500 bricks in a load weighing 4500 lb What is the weight of each brick in tenths of a pound? In ounces?

81. The mileage deducted from a mileage book for a party of 24 in traveling from one town to another was 528 miles. How far apart were the towns?

82. A machinist earns $46.20 per week, working eight hours a day for 5.5 days. How much does he get an hour?

83. If 51.6 tons of old iron were sold for $624.36, what was the price per ton? What was the price per pound? What would 1320 pounds bring?

84. John has 4 times as many marbles as James, and together they have 75 marbles. How many marbles has each?

85. Two boys cut a lawn for $1.20, sharing according to the time each worked. Henry worked three times as long as Joe did. How much pay should each boy receive?

86. How should three boys share $50 for a job where one boy does as much as both of the other two who do equal amounts of it?

87. Three children share a gift of $15 in proportion to their ages which are 1 year, 8 years, and 11 years. How much does each child receive?

II. Ratios

33. Ratios. The following set of exercises, some of which are solved, will illustrate the idea of ratios and the use of equations in problems that involve ratios.

EXERCISES

1. Compare 12 inches with 4 inches; 4 inches with 12 inches; 20 pounds with 5 pounds; 5 pounds with 20 pounds. Express each of the above comparisons as a fraction and reduce the fraction.

2. Compare 8 with 12.

SOLUTION. $\frac{8}{12} = \frac{2}{3}$; hence 8 is $\frac{2}{3}$ of 12.

That is, 8 has the same relation to 12 as 2 has to 3.

NOTE. This comparison of 8 to 12 or relation of 8 to 12, expressed by the fraction $\frac{8}{12}$, is called a ***ratio***.

3. Compare 12 with 8.

Here the comparison is expressed by the ratio $\frac{12}{8}$ or $\frac{3}{2}$.

4. What is the ratio of 15 to 20? Of 20 to 15? Of 10 to 16? Of 16 to 10?

5. Express the following ratios, reducing each to lowest terms: 18 to 10, 12 to 24, 6 to 9, 9 to 6, 11 to 17.

6. Express the ratios of the following quantities, reducing each ratio to the lowest terms:

(*a*) 18 in. to 24 in.
(*b*) 20 lb. to 70 lb.
(*c*) 35 ft. to 15 ft.
(*d*) 30 yd. to 36 yd.
(*e*) 42 lb. to 63 lb.
(*f*) 96 lb. to 108 lb.
(*g*) 48 da. to 60 da.
(*h*) 14 wk. to 49 wk.
(*i*) 12 oz. to 4 oz.
(*j*) 18 hr. to 9 hr.
(*k*) 4 oz. to 1 lb.
(*l*) 3 ft. to 5 yd.
(*m*) 3 yd. to 5 ft.
(*n*) 2 hr. to 45 min.
(*o*) 1760 ft. to 1 mi.
(*p*) 48 sq. in. to 1 sq. ft.
(*q*) $3\frac{1}{2}$ lb. to 8 oz.
(*r*) 144 cu. in. to 1 cu. ft.
(*s*) 75 cents to $3.25
(*t*) 5 qt. to 7 gal.

7. The ratio of a certain number to 6 is equal to $\frac{2}{3}$. Find the number.

SOLUTION. Let n = the number, then

$$\frac{n}{6} = \frac{2}{3}.$$

To solve the equation multiply each member by 6. The first member becomes

$$\frac{6\,n}{6} \text{ or } n.$$

The second member becomes

$$\frac{12}{3} \text{ or } 4.$$

Hence $n = 4.$

CHECK. $$\frac{4}{6} = \frac{2}{3}.$$

Ans. The number is 4.

8. The length of a room is 16 ft. The ratio of the width to the length is equal to $\frac{3}{4}$. Find the width of the room.

SOLUTION. Let w = number of feet in the width of room, then

$$\frac{w}{16} = \frac{3}{4}.$$

If we multiply each member by 16, the lowest common denominator of the two fractions, the first member becomes w, and the second member becomes 12. Hence we have

$$w = 12.$$

CHECK. $$\frac{12}{16} = \frac{3}{4}.$$

Ans. The width is 12 ft.

9. What number has the same ratio to 25 that 16 has to 100?

SOLUTION. Let n = the number, then

$$\frac{n}{25} = \frac{16}{100}.$$

Multiply each member by 100:

$$4\,n = 16.$$

Hence

$$n = 4.$$

CHECK.

$$\frac{4}{25} = \frac{16}{100}.$$

Ans. The number is 4.

10. Solve the equation $\frac{2\,n}{3} = 4$.

SOLUTION. The given equation is

$$\frac{2\,n}{3} = 4.$$

Multiply each member by 3:

$$2\,n = 12.$$

Divide each member by 2:

$$n = 6.$$

CHECK[1].

$$\frac{2 \times 6}{3} \stackrel{?}{=} 4.$$

$$\frac{12}{3} = 4.$$

Ans. $n = 6$.

[1] In checking, simplify each member by itself in order to avoid the possibility of repeating an error in your original solution. Retain the question mark over the equality sign in the check until the two members of the equation are shown to be equal. Read the sign $\stackrel{?}{=}$ "should equal".

11. Solve the equation $2 = \frac{4\,n}{5}$.

SOLUTION. The given equation is

$$2 = \frac{4\,n}{5}.$$

Multiply each member by 5:

$$10 = 4\,n.$$

Divide each member by 4:

$$2\tfrac{1}{2} = n.$$

CHECK. $$2 \stackrel{?}{=} \frac{4 \times 2\frac{1}{2}}{5}.$$

$$2 = \frac{10}{5}.$$

Ans. $n = 2\frac{1}{2}$.

12. Solve the equation $\frac{2\,n}{3} = \frac{4}{5}$.

SOLUTION. The given equation is

$$\frac{2\,n}{3} = \frac{4}{5}.$$

Multiply each member by 15:

$$10\,n = 12.$$

Divide each member by 10:

$$n = 1.2$$

CHECK. $$\frac{2 \times 1.2}{3} \stackrel{?}{=} \frac{4}{5}.$$

$$\frac{2.4}{3} \stackrel{?}{=} \frac{4}{5}.$$

$$.8 = .8$$

Ans. $n = 1.2$

(Each member must be simplified by itself.)

13. Solve the equation $\frac{r}{12} = \frac{3}{50}$.

SOLUTION. The given equation is

$$\frac{r}{12} = \frac{3}{50}.$$

Multiply each member by 300:

$$25\,r = 18.$$

Divide each member by 25:

$$r = .72$$

CHECK. $$\frac{.72}{12} \stackrel{?}{=} \frac{3}{50}.$$

$$.06 = .06$$

Ans. $r = .72$

NOTE. In solving Exercises 7–13, we have used the following axiom.

AXIOM. *If equal numbers are multiplied by the same number, the products are equal.*

Solve and check each of the following equations:

14. $\frac{y}{4} = \frac{11}{2}$.

15. $\frac{m}{4} = \frac{5}{2}$.

16. $\frac{a}{9} = \frac{7}{3}$.

17. $\frac{m}{10} = \frac{7}{15}$.

18. $\frac{p}{6} = \frac{5}{12}$.

19. $\frac{3}{4} = \frac{p}{3}$.

20. $\frac{r}{3} = \frac{5}{3}$.

21. $\frac{5\,r}{12} = \frac{5}{3}$.

22. $\frac{3\,b}{5} = \frac{2}{15}$.

23. $\frac{5}{6} = \frac{2\,w}{15}$.

24. $\frac{m}{5} = 7$.

25. $\frac{3\,m}{11} = 4$.

26. $\frac{5\,a}{2} = 8$.

27. $\frac{3\,b}{2} = .12$

28. $\frac{5\,x}{4} = .4$

29. $.25 = \frac{m}{4}$.

30. $\frac{7}{100} = \frac{m}{6}$.

31. $\frac{d}{4} = \frac{3}{100}$.

32. $\frac{3\,a}{4} = .09$

33. $\frac{5\,b}{12} = .2$

34. $\frac{4.2\,m}{3.6} = 2.1$

35. Out of a class of 50 pupils 40 were present. What was the number, or rate, per hundred present?

SOLUTION. This might be stated as follows. The ratio of 40 to 50 is equal to the ratio of what number to 100?

This statement would be expressed by the equation

$$\frac{40}{50} = \frac{n}{100},$$

where n = the number, or rate, per hundred; then

$$\frac{40}{50} = \frac{n}{100}.$$

Multiply each member by 100:

$$80 = n.$$

CHECK. $$\frac{40}{50} \stackrel{?}{=} \frac{80}{100}, \text{ or } .8 = .8$$

Ans. The rate per hundred is 80.

36. A boy pays 4 cents for 40 marbles. What is the price for 100?

SOLUTION. Let p = the number of cents per 100; then

$$\frac{4}{40} = \frac{p}{100},$$

or

$$\frac{1}{10} = \frac{p}{100}.$$

Multiply each member by 100:

$$10 = p.$$

CHECK. $$\frac{4}{40} \stackrel{?}{=} \frac{10}{100}, \text{ or } .1 = .1$$

Ans. The price per 100 is 10 cents.

37. In 1910 the population of a town was 2400. It increased 600 during the next 10 years. What was the rate of increase per hundred for the ten-year period?

38. One team in a baseball league has won 14 games out of 20 played. What is the team's rate of winning per hundred games?

39. In a class 9 out of every 10 pupils were present. The whole class numbered 70. How many were present?

SOLUTION. Let n = number of pupils present.

The ratio of 9 to 10 = the ratio of n to 70.

The equation is

$$\frac{9}{10} = \frac{n}{70}.$$

Multiply each member by 70:

$$63 = n.$$

CHECK. $\frac{9}{10} \stackrel{?}{=} \frac{63}{70}$, or $.9 = .9$

Ans. 63 pupils were present.

40. At 45 cents a dozen how much will 16 oranges cost? (The ratio of 16 to 12 = ratio of what number to 45?) Form the equation and solve it.

41. Bananas are selling 15 for 50 cents. How much are they per dozen? (Be sure to arrange your statement so that the required number is the *first* number of one ratio.)

42. Lemons are selling 3 for 10 cents. What will $1\frac{1}{2}$ dozen cost?

43. Tumblers are selling at 70 cents a dozen. What will 9 tumblers cost?

44. If 4 apples cost 15 cents, what will 10 apples cost?

45. If soap is selling at 6 bars for a quarter, what will 4 bars cost? (Answer to nearest cent.)

46. Post cards are sold at 80 for $1.00. At this rate what will 100 cards cost?

47. American League baseballs are sold at $15.00 a dozen. Find the cost of 5 balls.

48. If 25 yards of cloth cost $7.50, how many yards can be bought for $60?

49. A school contains 1800 pupils; 4 out of every 9 are foreign-born. How many pupils are foreign-born?

50. An airplane flies 375 miles in $2\frac{1}{2}$ hours. At the same speed how far will it fly in 4 hours?

51. On a certain map a line $1\frac{1}{2}$ inches long represents a distance of 120 miles. What distance is represented by a line 5 inches long?

52. At $9.60 a hundred pounds, find the cost of 275 pounds of lamb.

53. If 50 feet of garden hose sell for $8.00, what will 125 feet cost?

54. At $15.00 a ton of 2000 pounds, what will 7000 pounds of coal cost?

55. The ratio $\frac{3}{4}$ is what rate per hundred?

SOLUTION.

$$\frac{3}{4} = \frac{n}{100}.$$

Multiply each member by 100:

$$75 = n.$$

CHECK.

$$\frac{3}{4} = \frac{75}{100}.$$

Ans. The rate per hundred is 75.

56. The ratio $\frac{1}{8}$ is what rate per hundred?

57. $\frac{3}{8}$ = how many per hundred?

58. $\frac{7}{8}$ = how many per hundred?

59. $\frac{5}{8}$ = how many per hundred?

60. $\frac{1}{3}$ = how many per hundred?

61. $\frac{2}{3}$ = how many per hundred?

62. $\frac{1}{5}$ = how many per hundred?

63. $\frac{2}{5}$ = how many per hundred?

64. $\frac{3}{5}$ = how many per hundred?

65. $\frac{4}{5}$ = how many per hundred?

66. $\frac{1}{6}$ = how many per hundred?

67. $\frac{5}{6}$ = how many per hundred?

68. $\frac{1}{12}$ = how many per hundred?

69. $\frac{1}{16}$ = how many per hundred?

70. Copy and fill in the following table:

Ratio	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{5}{6}$
Number per hundred	50										

71. Copy and fill in the following table:

Common fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{5}{6}$
Decimal fraction expressed in hundredths				.12$\frac{1}{2}$							

III. Percentage

34. In the Exercises 55–71 on the last two pages you have learned how to find the rate per hundred or the number per hundred corresponding to any given ratio.

Per cent means the number per hundred. Thus 96 per cent (written 96%) means 96 per hundred. For example, 96% attendance means that 96 of every 100 pupils are present.

Example 1. In a spelling test of 100 words,

John spelled 96 correctly, or 96 per 100, or 96%;
Mary spelled 92 correctly, or 92 per 100, or 92%;
James spelled 88 correctly, or 88 per 100, or 88%;
Henry spelled 84 correctly, or 84 per 100, or 84%.

As there were 100 words in this test and per cent means per hundred, the number of words spelled correctly by each pupil gives also the per cent obtained by that pupil.

Example 2. In a spelling test of 25 words,

John spelled 24 correctly, or $\frac{24}{25}$ or $\frac{96}{100}$, or 96%;
Mary spelled 23 correctly, or $\frac{23}{25}$ or $\frac{92}{100}$, or 92%;
James spelled 22 correctly, or $\frac{22}{25}$ or $\frac{88}{100}$, or 88%;
Henry spelled 21 correctly, or $\frac{21}{25}$ or $\frac{84}{100}$, or 84%.

John spelled correctly at the rate of 24 per 25 or 96 per 100; that is, John spelled 96% of the words correctly.

Mary spelled correctly 23 of 25 (or at the rate of 92 per 100) and so spelled 92% (of the words) correctly.

James spelled correctly 22 of 25 (or 88 per 100), or 88%.
Henry spelled correctly 21 of 25 (or 84 per 100), or 84%.

EXERCISES

1. If 20 words were given, and

John spelled correctly 18 words,
Mary spelled correctly 19 words,
James spelled correctly 15 words, and
Henry spelled correctly 14 words,

then what per cent of the words did each spell correctly? What per cent of the words did each misspell?

(John missed 2 out of 20 words, or at the rate of 10 per 100, or 10%.)

2. Ten examples were given in a speed test and only correct answers counted. John had 6 correct, Mary 9, James 7, Henry 8. What per cent did each obtain on this test?

3. In a series of 5 games, team A won 3 games and team B, 2 games. What per cent of the games did each win?

35. *Per cent* may also be called the number of hundredths of some given amount. For example, if a line AB

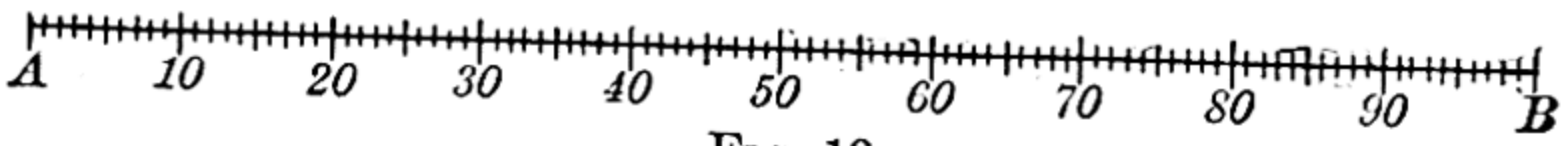


FIG. 10

of definite length be divided into 100 equal parts, as shown in Fig. 10, then

10 of these parts (10 of 100) represent .10 of the length of the line, or 10% of it;

20 parts represent .20 of the line, or 20% of it;

60 parts represent .60 of the line, or 60% of it;

85 parts represent .85 of the line, or 85% of it;

$33\frac{1}{3}$ parts represent $.33\frac{1}{3}$ of the line, or $33\frac{1}{3}\%$ of it.

36. If our foot rule is divided into ten equal parts (tenths), then one of these parts is $\frac{1}{10}$ of a foot, or $\frac{10}{100}$ of a foot, or 10% of a foot,

2 of these parts, $\frac{20}{100}$ of a foot, or 20% of a foot,

4 of these parts, $\frac{40}{100}$ of a foot, or 40% of a foot.

If our inch is subdivided into *tenths*, express the per cent that one tenth is of the inch. Do this for .2 in., .3 in., etc.

37. The relation that any line bears to a second line may be expressed as a per cent relation, for the ratio that the first line bears to the second may be expressed as a ratio per hundred.

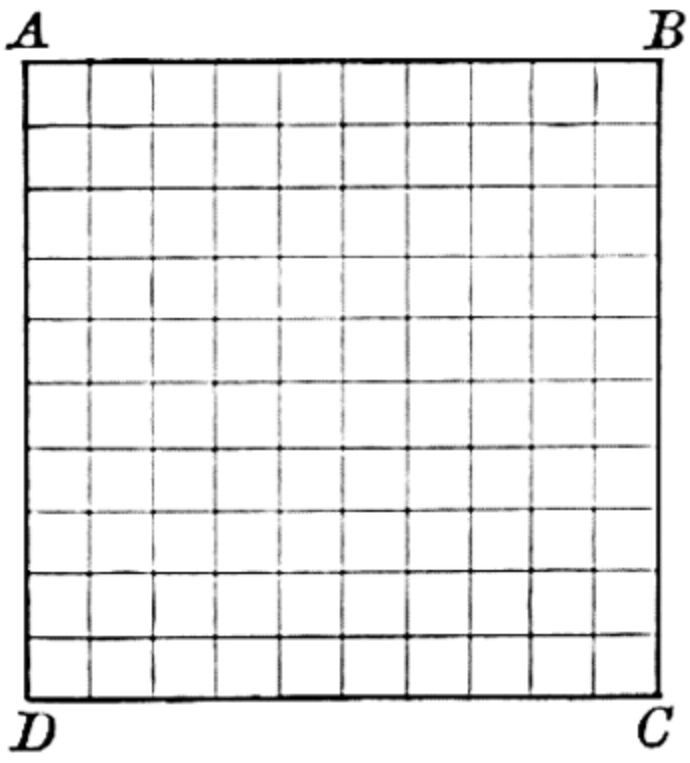


FIG. 11

Thus, if a line $AB = 4.5$ in., and a line $CD = 5.0$ in., then,

$$\frac{AB}{CD} = \frac{4.5}{5.0} = \frac{90}{100};$$

that is,

line AB is 90% of line CD.

38. If we have a square $ABCD$ subdivided into 100 equal squares as shown (Fig. 11), then

1 of the small squares = $\frac{1}{100}$, or .01, or 1% of the large square;

2 of the small squares = $\frac{2}{100}$, or .02, or 2% of the large square;

6 of the small squares = .06, or 6% of the large square;

1 row of small squares = .10, or 10% of the large square.

2 rows of small squares = what % of the large square?

3 rows of small squares = what % of the large square?

4 rows of small squares = what % of the large square?

8 rows of small squares = what % of the large square?

EXERCISES

1. What per cent of a foot is 6 inches? 4 inches? 9 inches? 12 inches? 1 inch? 18 inches?

2. What per cent of a yard is 18 inches? 9 inches? 27 inches? 36 inches? 1 foot? 45 inches?

3. What per cent of a square foot is 72 square inches? 36 square inches? 108 square inches? 144 square inches?

4. The following table represents the results of a test in arithmetic. Fill in the values required.

Student	Number of Examples	Number Right	Number Wrong	Per Cent Right	Per Cent Wrong
a.	10	8	?	?	?
b.	10	7	?	?	?
c.	10	6	?	?	?
d.	10	5	?	?	?
e.	10	?	4	?	?
f.	10	?	3	?	?
g.	10	?	?	?	50
h.	10	?	?	30	?

5. The following table represents the results of a test in spelling. Fill in the values required.

Student	Number of Words	Number Correct	Number Incorrect	Per Cent Correct	Per Cent Incorrect
a.	20	17	?	?	?
b.	25	21	?	?	?
c.	50	?	7	?	?
d.	25	22	?	?	?
e.	50	?	16	?	?
f.	50	?	?	80	?
g.	25	?	?	60	?
h.	20	?	?	?	10

6. In your proficiency tests using Tables I, IV, and V, etc., keep a record now of the number of exercises attempted as well as the number correct.

For each table on each test:

(*a*) Find what per cent the number correct is of the number attempted.

(*b*) Find what per cent the number incorrect is of the number attempted.

(*c*) Draw the graphs of these percentages for successive tests on the same sheet of paper. Try to attain 100%.

39. The original amount is commonly regarded as 100% (of itself). Thus, in Exercise 4, to get 100% on the test, a pupil must get 10 examples right.

40. It is important to notice that per cent always refers to per cent (or number of hundredths) of *some definite amount*. Frequently the definite amount is not stated. For example, per cents obtained by students in tests, such as 95%, 92%, 85%, etc., do not necessarily tell us how many examples or questions were given. Such per cents, however, do give us a means of comparing the results.

EXERCISES

1. What per cent of 25 is 3? 12? 17? 23? 25? 28? 50?

SOLUTION. These exercises can be done orally if you can think through the following steps. In the more difficult ones use the equation method. (Ex. 55, p. 84.)

ORAL METHOD

$\frac{3}{25} = \frac{?}{100} = ?\ \%.$

$\frac{3}{25} = \frac{12}{100} = 12\%.$

CHECK. $\frac{3}{25} = \frac{12}{100}.$

EQUATION METHOD

$\frac{3}{25} = \frac{n}{100}.$

Multiply each member by 100:

$12 = n.$

Ans. 3 is 12% of 25.

2. What per cent of 50 is 10? 18? 26? 37? 50? 57?

3. What per cent of 300 is 27? 36? 54? 90? 129? 210? 300? 315? 420? 600? 135? 87? 9? 3?

4. What per cent of 400 is 28? 120? 260? 300? 380? 400? 800? 520? 68? 52? 24? 8? 4?

5. What per cent of 600 is 6? 36? 60? 72? 240? 300? 420? 600? 84? 24? 720? 624? 42? 528?

41. FRACTIONAL PARTS OF THE WHOLE EXPRESSED AS HUNDREDTHS AND AS PER CENTS

$\frac{1}{10} = .10 = 10\%$	$\frac{3}{5} = .60 = 60\%$
$\frac{1}{8} = .12\frac{1}{2} = 12\frac{1}{2}\%$	$\frac{3}{4} = .75 = 75\%$
$\frac{1}{6} = .16\frac{2}{3} = 16\frac{2}{3}\%$	$\frac{4}{5} = .80 = 80\%$
$\frac{1}{5} = .20 = 20\%$	$\frac{2}{3} = .66\frac{2}{3} = 66\frac{2}{3}\%$
$\frac{1}{4} = .25 = 25\%$	$\frac{3}{8} = .37\frac{1}{2} = 37\frac{1}{2}\%$
$\frac{1}{3} = .33\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{5}{8} = .62\frac{1}{2} = 62\frac{1}{2}\%$
$\frac{2}{5} = .40 = 40\%$	$\frac{7}{8} = .87\frac{1}{2} = 87\frac{1}{2}\%$
$\frac{1}{2} = .50 = 50\%$	$\frac{5}{6} = .83\frac{1}{3} = 83\frac{1}{3}\%$

The above fractions are so commonly used that their equivalent per cents should be memorized.

EXERCISES

1. What per cent of 8 is 6?

SOLUTION. These exercises can be done without paper and pencil if you can think through the following steps:

ORAL METHOD

$6 = \frac{6}{8}$ of 8, or $\frac{3}{4}$ of 8.

$\frac{3}{4} = \frac{?}{100} = ?\%$.

CHECK. $\frac{3}{4} = \frac{75}{100}$.

EQUATION METHOD

$\frac{3}{4} = \frac{n}{100}$.

$75 = n$.

Ans. 6 is 75% of 8.

2. What per cent of 24 is 20?

SOLUTION. $20 = \frac{20}{24}$ of 24, or $\frac{5}{6}$ of 24.

ORAL METHOD

$\frac{5}{6} = \frac{?}{100}$.

CHECK. $\frac{5}{6} = \frac{83\frac{1}{3}}{100} = 83\frac{1}{3}\%$.

EQUATION METHOD

$\frac{5}{6} = \frac{n}{100}$.

$83\frac{1}{3} = n$.

Ans. 20 is $83\frac{1}{3}\%$ of 24.

3. In Fig. 12, the number at the center is what per cent of each number on the rim?

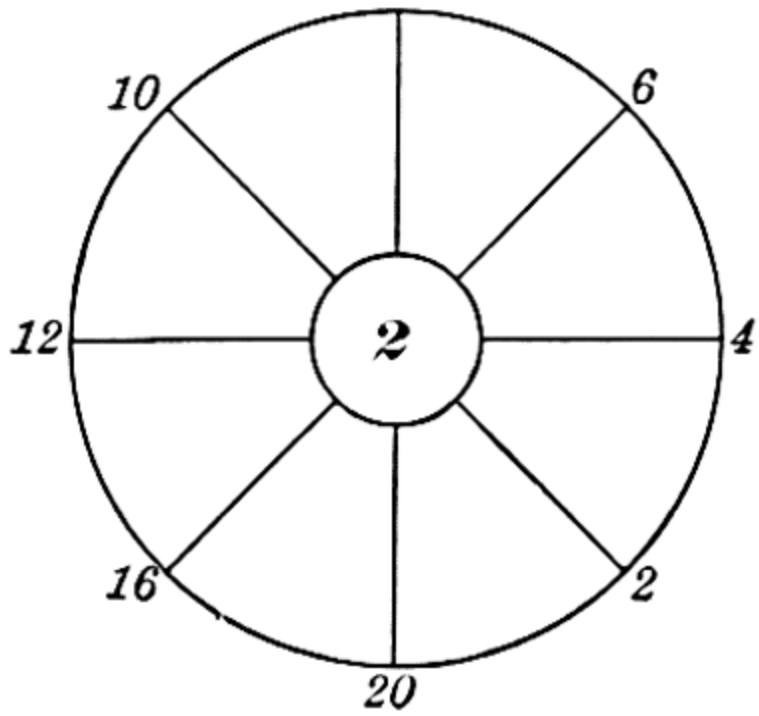


FIG. 12

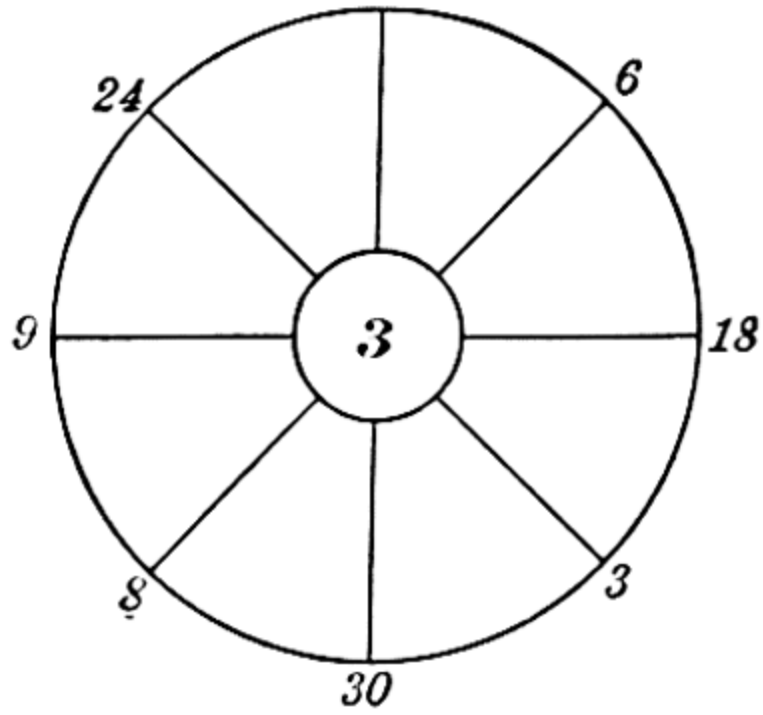


FIG. 13

Begin as follows:

2 is $\frac{1}{4}$ of 8, or 2 is 25% of 8.

2 is $\frac{1}{5}$ of 10, or 2 is 20% of 10.

In this manner use each number on the rim.

4. In Fig. 13, the number at the center is what per cent of each number on the rim?

5. In Fig. 14, the number at the center is what per cent of each number on the rim?

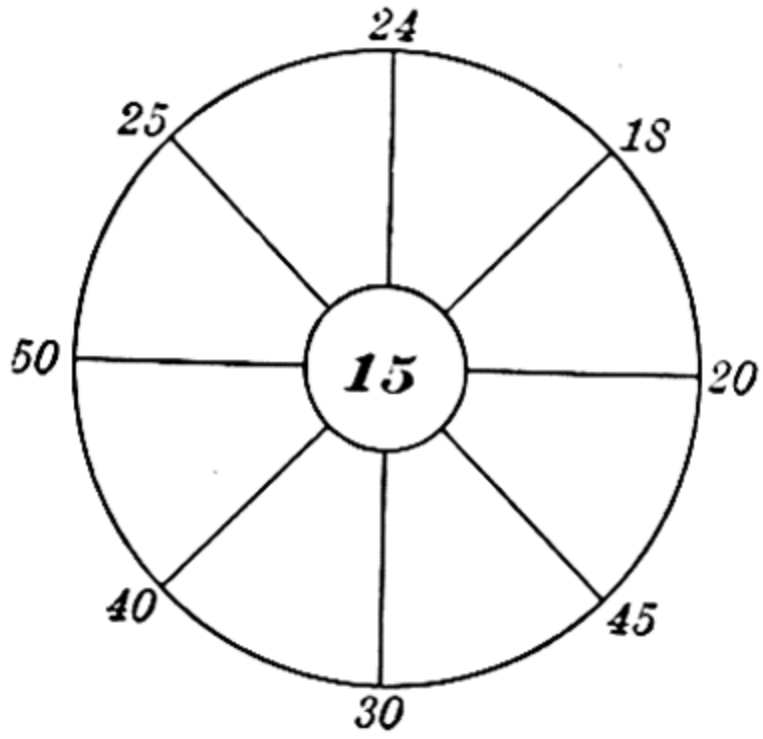


Fig. 14

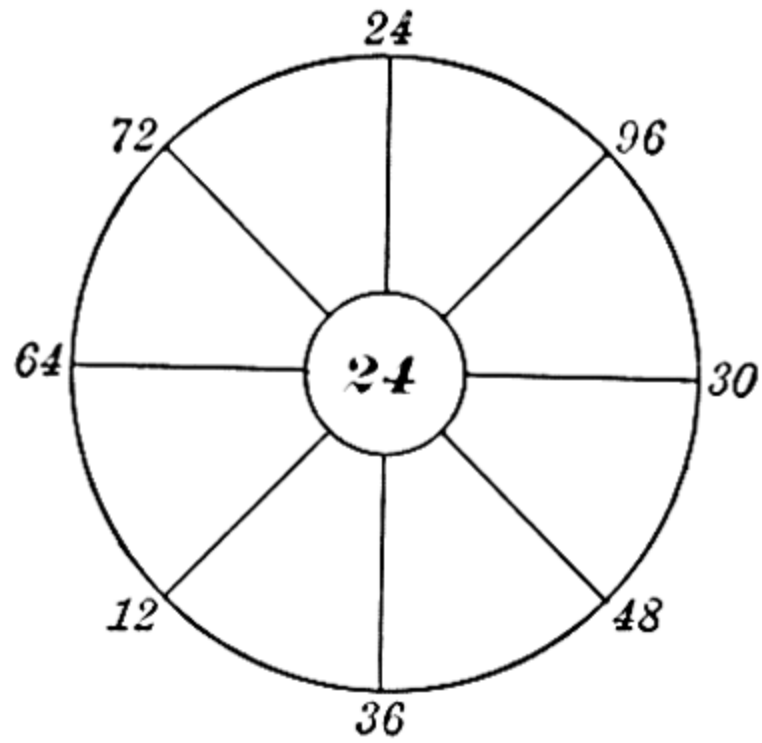


Fig. 15

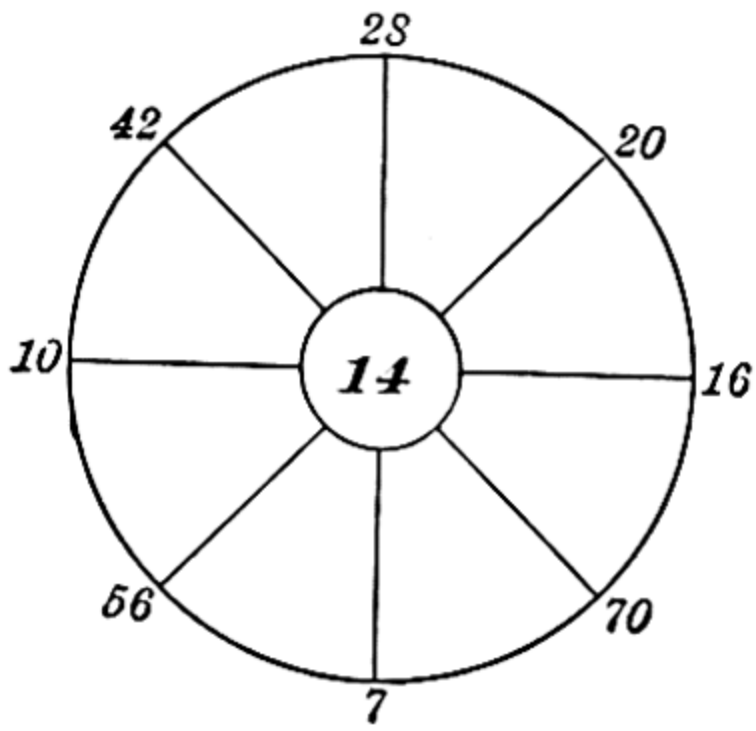


Fig. 16

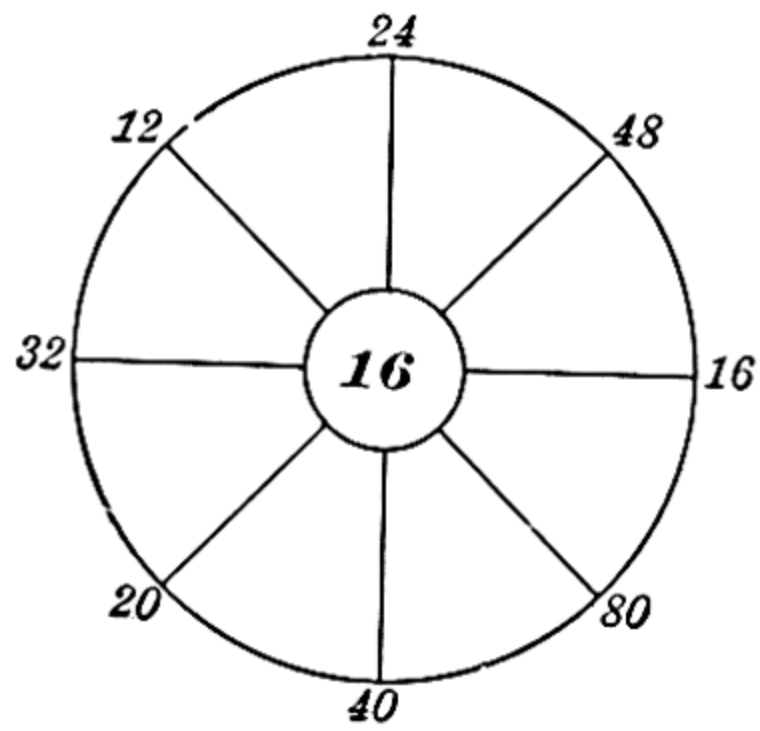


Fig. 17

6. In Fig. 15, the number at the center is what per cent of each number on the rim?

7. In Fig. 16, the number at the center is what per cent of each number on the rim?

8. In Fig. 17, the number at the center is what per cent of each number on the rim?

9. What per cent of 64 is 16? 32? 48? 8? 24? 40? 56? 64? 96? 80? 112? 128?

10. What per cent of 56 is 14? 28? 7? 21? 35? 56? 70? 84? 98? 112? 168? 280?

11. What per cent of 96 is 24? 32? 48? 64? 96? 120? 16? 80? 8? 40? 144? 288?

12. What per cent of 120 is 40? 48? 72? 96? 100? 140? 60? 240? 80? 90? 180? 600?

13. What per cent of 72 is 36? 18? 24? 27? 45? 54? 63? 72? 9? 90? 128? 144?

14. What per cent of 144 is 16? 48? 72? 108? 120? 132? 180? 288? 18? 90? 720? 1440?

15. What per cent of 32 is 15?

SOLUTION. $15 = \frac{15}{32}$ of 32.

ORAL METHOD	EQUATION METHOD
$\frac{15}{32} = \frac{?}{100}$.	$\frac{15}{32} = \frac{n}{100}$.

Multiply each member by 800 (the lowest common denominator of 32 and 100):

$$375 = 8\,n.$$

Divide each member by 8:

$$n = 46\tfrac{7}{8}.$$

CHECK. $46\frac{7}{8}\%$ of 32 = 15.

Ans. $15 = 46\frac{7}{8}\%$ of 32.

NOTE. Instead of multiplying each member by 800, in the first step, you could multiply each member by 100. This would give

$$\frac{1500}{32} = n.$$

The value of n then would be found by the process of long division. A third way would be to express the value of n in ***hundredths*** by finding the decimal equivalent of the common fraction $\frac{15}{32}$. The number of hundredths would be the per cent. The method of long division is shown in the next exercise.

16. What per cent of 32 is 22? 31? 32? 38? 58?

SOLUTION. $22 = \frac{22}{32}$ of 32, or $\frac{11}{16}$ of 32.

$$\frac{11}{16} = \frac{n}{100}.$$

WORK

Multiply each member by 100:

$$\frac{1100}{16} = n,$$

$$n = 68\tfrac{3}{4}.$$

$$\begin{array}{r} 68\frac{3}{4} \\ 16\overline{)1100} \\ \underline{96} \\ 140 \\ \underline{128} \\ 12 \end{array}$$

$$\frac{12}{16} = \frac{3}{4}$$

CHECK. $68\frac{3}{4}\%$ of 32 is 22.

Ans. $22 = 68\frac{3}{4}\%$ of 32.

NOTE. This answer, *to the nearest per cent*, is 69% (since $\frac{3}{4}$ is more than $\frac{1}{2}$). After obtaining the answers in each of the Exercises 17 to 19, write the per cents to the nearest per cent.

17. What per cent of 17 is 5? 8? 11? 15? 21?

18. What per cent of 42 is 40? 41? 39? 2? 3?

19. What per cent of 125 is 123? 12? 118?

APPLIED PROBLEMS

1. A farmer raised 80 bushels of potatoes on an acre of land in 1921. In 1922 he raised 90 bushels. What was the increase? What was the per cent of increase?

SOLUTION.

90 (bu.) in 1922
80 (bu.) in 1921
10 (bu.) increase

$\frac{10}{80}$ = what per cent (*i.e.* 10 = what per cent of 80)?
$\frac{10}{80} = \frac{1}{8} = 12\frac{1}{2}\%$.
Hence 10 bushels is $12\frac{1}{2}\%$ of 80 bushels.
Ans. The increase was 10 bushels; the per cent of increase was $12\frac{1}{2}\%$.

2. John paid 50 cents for 100 papers. He sold them for $1. What was his gain? What was his gain per cent?

3. In a room of 40 pupils 12 are absent. What is the percentage of absence? Of attendance?

4. In a series of 5 games team A wins 4. What is the per cent of games won?

5. In a room of 42 pupils 28 are girls. What per cent of the pupils are girls?

6. In a school of 640 pupils 420 are boys. What per cent of the pupils are boys?

7. A clerk whose weekly salary is $24 saves $5. What per cent of his salary does he save?

8. A carload of wheat costs a grain dealer $1750. He sells it for $2000. What is his profit? What is his per cent of profit on the cost?

9. A fruit dealer pays $3.00 for a bunch of bananas. He sells from this bunch $8\frac{1}{2}$ dozen at 30 cents a dozen and 5 dozen at 25 cents a dozen. How much does he get for

the bananas sold? What is his profit? What is his per cent of profit on the cost?

10. What is gained if a coat costing \$6.40 is sold for \$10? What per cent is gained on the cost?

11. A's salary in 1921 was \$1500. His salary in 1922 was \$1650. What was the increase? What was the per cent of increase?

12. The average yield of wheat per acre in the United States in 1921 was 12.8 bushels; in 1922, 14.0 bushels. What was the increase? What was the per cent of increase?

13. The average yield of potatoes in the United States per acre in 1921 was 90.9 bushels; in 1922, 102.6 bushels. What was the per cent of increase?

14. The average yield of corn per acre in 1920 was 29.6 bushels; in 1921, 28.2 bushels. What was the decrease? What was the per cent of decrease?

15. The average yield of oats per acre in 1921 was 23.7 bushels; in 1922, 29.9 bushels. What was the increase? What was the per cent of increase?

16. The price of illuminating gas, in a certain city, was recently reduced from \$1.25 per 1000 cubic feet to \$1.20. What was the per cent of reduction?

17. The maximum wage of motormen was recently raised from 61 cents per hour to 70 cents per hour. What was the per cent of increase?

18. The price of gasoline was recently raised from 17 cents per gallon to 19 cents per gallon. What was the per cent of increase?

19. Lime cost the farmer \$2.97 per barrel in 1922. The price in 1909 was \$1.29; in 1914, \$1.36; in 1920, \$3.10; in 1921, \$2.65. What has been the per cent of increase or decrease in 1922 on the cost in each of the other years?

QUESTIONS FOR REVIEW

1. What two operations can you perform on both sides of an equation without destroying the equality?

2. Is there any difference in the solution of the equation $18 = 3y$ and the equation $3a = 18$?

3. Does the letter used determine the answer of an equation? Explain.

4. Does it make any difference on which side of the equality sign the letter is placed in an equation?

5. In the process of checking an equation the letter does not appear in the work. What becomes of it?

6. What is the ratio of 475 to 750? Express it in two ways. What is the ratio of 750 to 475?

7. Is it possible to express all the following as ratios? The ratio of 12 books to 8 books; 12 books to 8 pencils; 12 feet to 8 yards; 12 examples to 4 grades.

8. What similarity exists between numbers that have a ratio?

9. If a boy had 7 problems right out of 9 attempts and a girl had 9 right out of 11 attempts, which had the greater per cent right of those each tried?

10. Write the per cents corresponding to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$. (Do not refer to your book.)

11. How many are $5a + 6a + 3a + 7a$? In adding such quantities as these do you add the letters or the numbers preceding the letters?

12. The total weight of two boys is 156 pounds. One boy weighs $16\frac{2}{3}$ per cent more than the other boy. Can you find the weight of each boy?

CHAPTER V

MEASUREMENT

I. Lines and Angles

42. Lines and Planes. Many of the surfaces about you are flat; for example, the blackboards, the top of your desk, the floor, the walls. These surfaces are called ***plane surfaces*** or ***planes***. If you draw a line on one of these surfaces with the aid of your ruler, you say the line is straight, which means that it has the same direction, however long you make it.

These are tools used by the carpenter to plane boards.

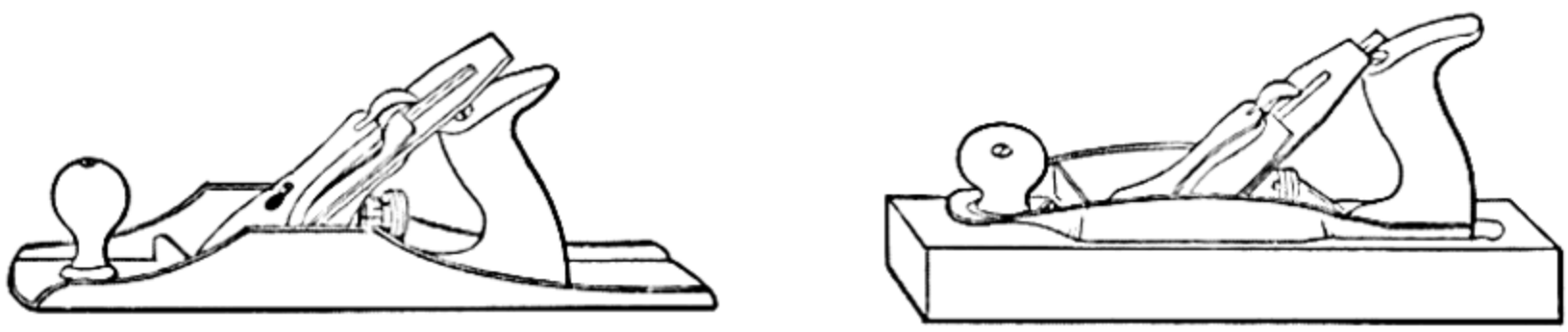


Fig. 18

Table of Linear Measure

12 inches (in.) = 1 foot (ft.)

3 feet = 1 yard (yd.)

$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet = 1 rod (rd.)

320 rods or 5280 feet = 1 mile (mi.)

EXERCISES

[For the work that follows you should have a ruler graduated in sixteenths of an inch on one edge and tenths of an inch on another. For this and other constructions your pencil should be sharpened to a fine point.]

1. To test the straightness of the edge of your ruler, mark two points on a piece of paper. Using your ruler, draw a line through them. Turn the ruler around and again draw a line, along the *same edge of the ruler*, through the same two points. If the ruler has a straight edge, only one line will be visible.

From this construction you observe that two points are necessary to show the direction of a straight line.

2. To determine whether a surface is plane or not, place the edge of your ruler against the surface to be tested. If the edge touches the surface at all points for every possible position of the ruler, the surface is plane. Test the top of your desk, the blackboards, and other objects that you think have plane surfaces.

3. Estimate the distance from one end of your classroom to the other by first measuring the length of your "step" and then by counting the number of steps which you take in going this distance. This method of measuring is called *pacing*.

4. Estimate the length and width of various objects in your classroom. If you were to measure these distances, tell in each case which of the following would be the best measuring stick to use: 6-inch rule, foot rule, yardstick, 60-inch tape, 50-foot tape, 100-foot tape.

5. Several straight lines are shown in Fig. 19. We shall read these lines by the two letters at their ends,

for example, AB, meaning the straight line AB. From now on; the word *line* will stand for a *straight line.*

Measure each line in Fig. 19 to the nearest tenth of an inch. Keep a record of the length of each line as follows: $AB = ?$ in., $CD = ?$ in., etc.

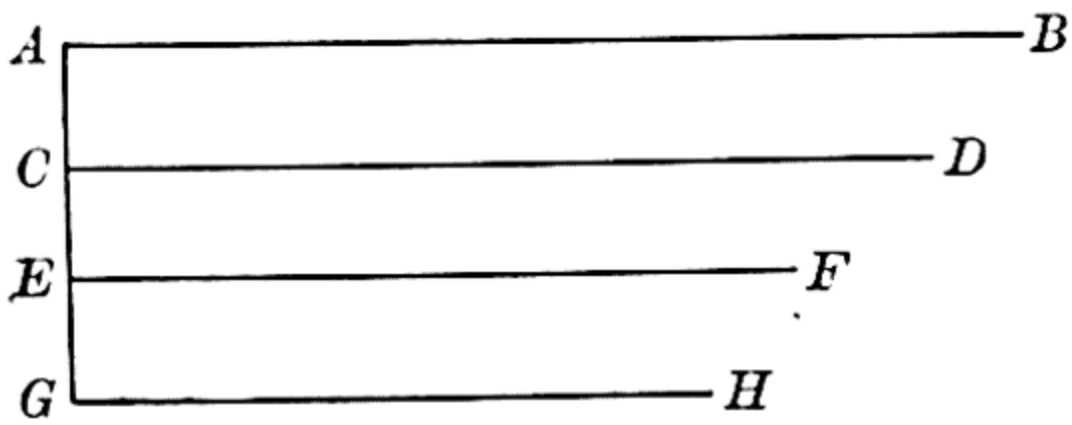


FIG. 19

6. Using the measurements just obtained, answer the following questions:

$AB - CD = ?$	$EF - GH = ?$
$AB - EF = ?$	$GH + ? = CD$
$AB - GH = ?$	$EF + ? = AB$
$CD - EF = ?$	$GH + ? = EF$
$CD - GH = ?$	$GH + ? = AB$

7. Measure each line in Fig. 19 to the nearest *eighth* of an inch, and keep a record of these lengths as above. Using these measurements, answer the questions in Exercise 6.

8. Can you find the answers to the questions in Exercises 6 and 7 by using the ruler alone? Explain how you do it.

9. Find the average of the lengths of the four lines in Fig. 19 by using the measurements (*a*) to tenths of an inch and (*b*) to eighths of an inch.

10. Using the measurements to tenths of an inch, answer the following questions:

$$\frac{AB}{CD} = \frac{2.1 \text{ in.}}{1.9 \text{ in.}} = \frac{21}{19}. \qquad \frac{CD}{EF} = \frac{?}{?}$$

$$\frac{AB}{EF} = \frac{?}{?} \qquad \frac{CD}{GH} = \frac{?}{?}$$

$$\frac{AB}{GH} = \frac{?}{?} \qquad \frac{EF}{GH} = \frac{?}{?}$$

The ratio of AB to CD is 21 to 19, or the length of AB is 1.1 times that of CD.

11. Using the measurements to eighths of an inch, answer the same questions in Exercise 10. Why do not these answers exactly agree with those in Exercise 10?

12. Measure the length and width of this page. What is the ratio of its length to its width? What is the ratio of its width to its length? What per cent of the length is the width?

13. Estimate the dimensions of the blackboard. Select a suitable measuring stick and measure its length and height. What is the ratio of its length to its height? What per cent of the length is the height?

14. Draw a straight line $\frac{5}{8}$ of an inch long. Compare it with a line .6 of an inch long. With a line .7 of an inch long. Tell which is the longest.

15. Draw a straight line $1\frac{7}{8}$ inches long. Compare it with a line 1.8 inches long. With a line 1.9 inches long.

16. Draw three straight lines of different lengths. Measure each to the nearest tenth of an inch. Add the lengths and divide the sum by 3 to get the average of the lengths of the three lines.

17. Draw four straight lines of different lengths. Measure each to the nearest tenth of an inch. Find the average of the lengths of the four lines by adding them and dividing by 4. Measure the same four lines each to the nearest eighth of an inch. As before, find the average of the lengths of the four lines by adding and dividing by 4.

18. Draw five straight lines and measure as above. Find the average of the lengths.

19. Draw two straight lines of different lengths. Measure them to the nearest tenth of an inch. Find their ratio.

20. Draw a straight line. Measure it to the nearest eighth of an inch. Measure it to the nearest tenth of an inch. Change your measurement in eighths to a decimal and compare it with the measurement in tenths.

43. Angles. When a straight line starting from a point is turned about that point from one position to another, an *angle* ($\angle$) is formed by the two positions of the line. The greater the amount of turning the greater the angle. For example, the hands of a clock form different angles with each other as they move from minute to minute.

44. In Fig. 20 the line is turned about O from the position OA to the position OB, forming the angle AOB. O is called the *vertex* of the angle AOB, and OA and OB the *sides*.

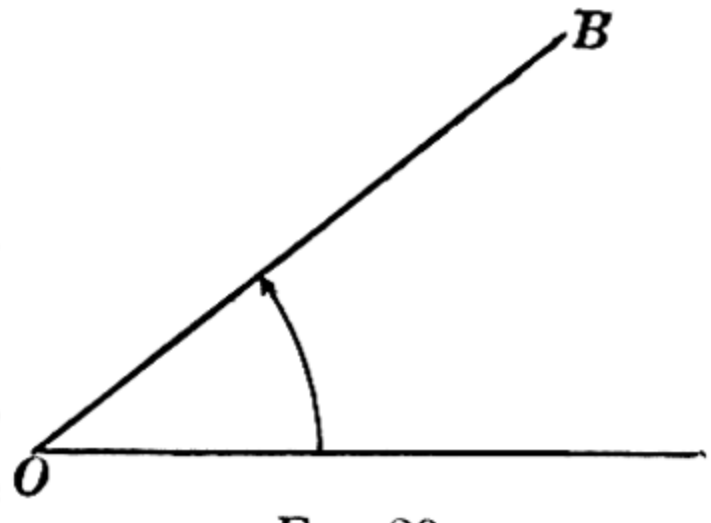


FIG. 20

In reading an angle the vertex letter is always read in the middle ($\angle$ AOB). Therefore, an angle like the one in Fig. 20 may be read angle AOB or angle BOA.

45. If the line is turned about O from the position OA unti. it again takes the position OA, a ***revolution*** is completed A revolution is divided into 360 equal parts called ***degrees*** (360°). A degree is divided into 60 equal parts called ***minutes*** (60′). A minute is divided into 60 equal parts called ***seconds*** (60″).

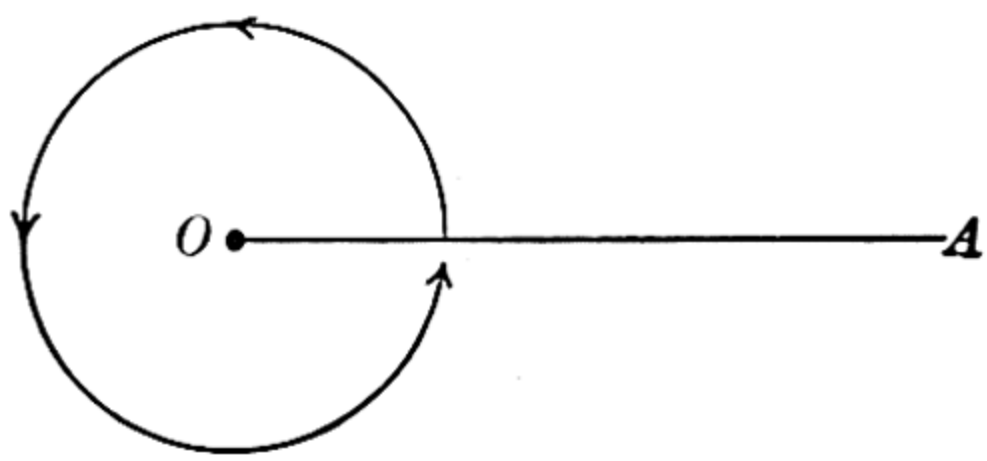


FIG. 21

TABLE OF DEGREE MEASURE

60 seconds (″) = 1 minute (′)

60 minutes = 1 degree (°)

360 degrees = 1 revolution

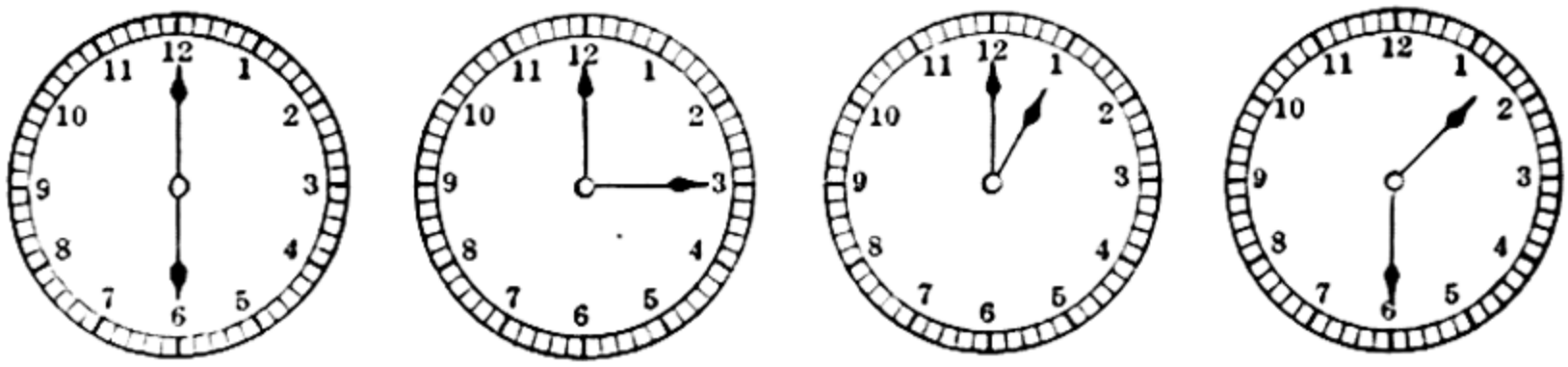


FIG. 22

The hands of a clock at 6 o'clock form an angle equal to one half a revolution, or 180°. At 3 o'clock they form an angle of 90°; at 1 o'clock they form an angle of 30°; at 1:30 o'clock they form an angle of 135° (180° − 45°).

EXERCISES

1. What angles are made by the hands of the clock at 2 o'clock? At 4 o'clock? At 5 o'clock? At 7 o'clock?

2. What angles are made by the hands of the clock at

2:30 o'clock? At 3:30 o'clock? At 4:30 o'clock? At 5·30 o'clock?

3. What angles are made by the hands of the clock at 1:15 o'clock? At 2:15 o'clock? At 4:15 o'clock?

One instrument used for measuring angles is called a *protractor*. (See Fig. 23.) For the following exercises the student should have this or some other form of protractor.

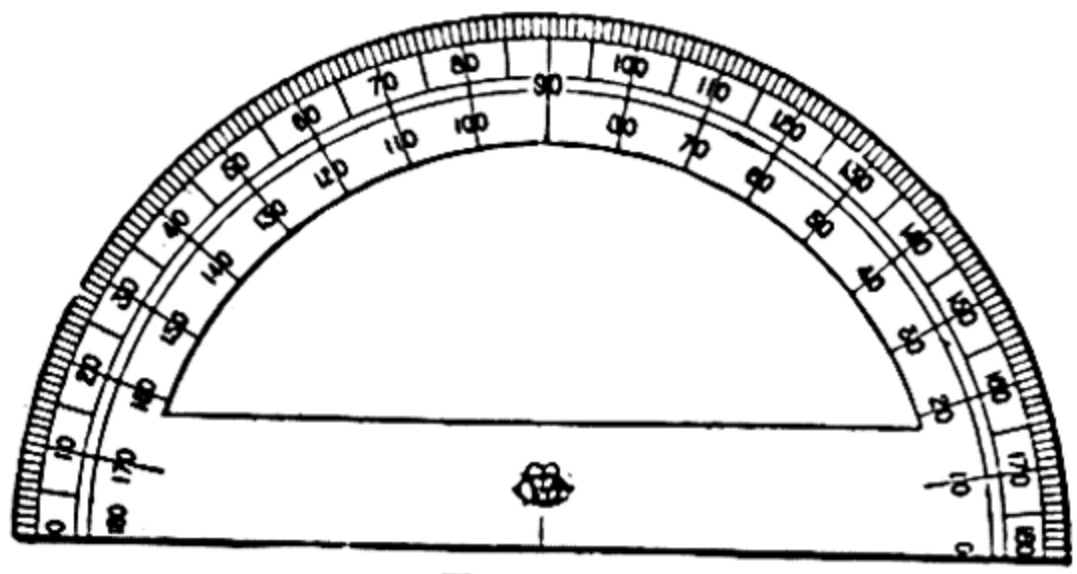


FIG. 23

EXERCISES

1. With the ruler draw an angle of 90°. Test the accuracy of it with your protractor.

2. With the ruler draw an angle of 45° (one half of 90°). Also draw an angle of 30°; of 60°; of 120°; of 150°. Test the accuracy of each angle by measuring it with your protractor. Practice drawing these angles without your protractor until you become fairly accurate.

3. Measure the angle in Fig. 24 with your protractor. Is it necessary that the sides of the angle be of equal

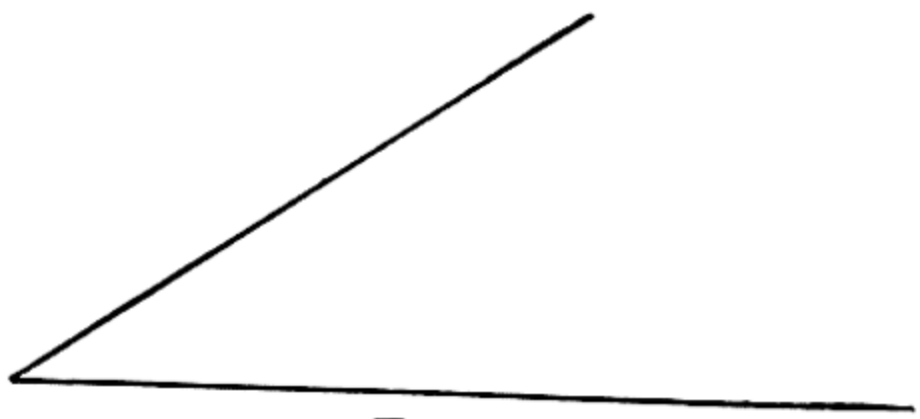


FIG. 24

length? Does the size of an angle depend upon the lengths of its sides? What determines the size of an angle? (See §43.)

4. Using the ruler only, draw an angle of 225°; of 210°; of 240°. Test the accuracy of each angle by measuring it with the protractor.

46. A *right angle* is one fourth of a revolution, or an angle of 90°. At 9 o'clock and at 3 o'clock the hands of a clock are at right angles to each other.

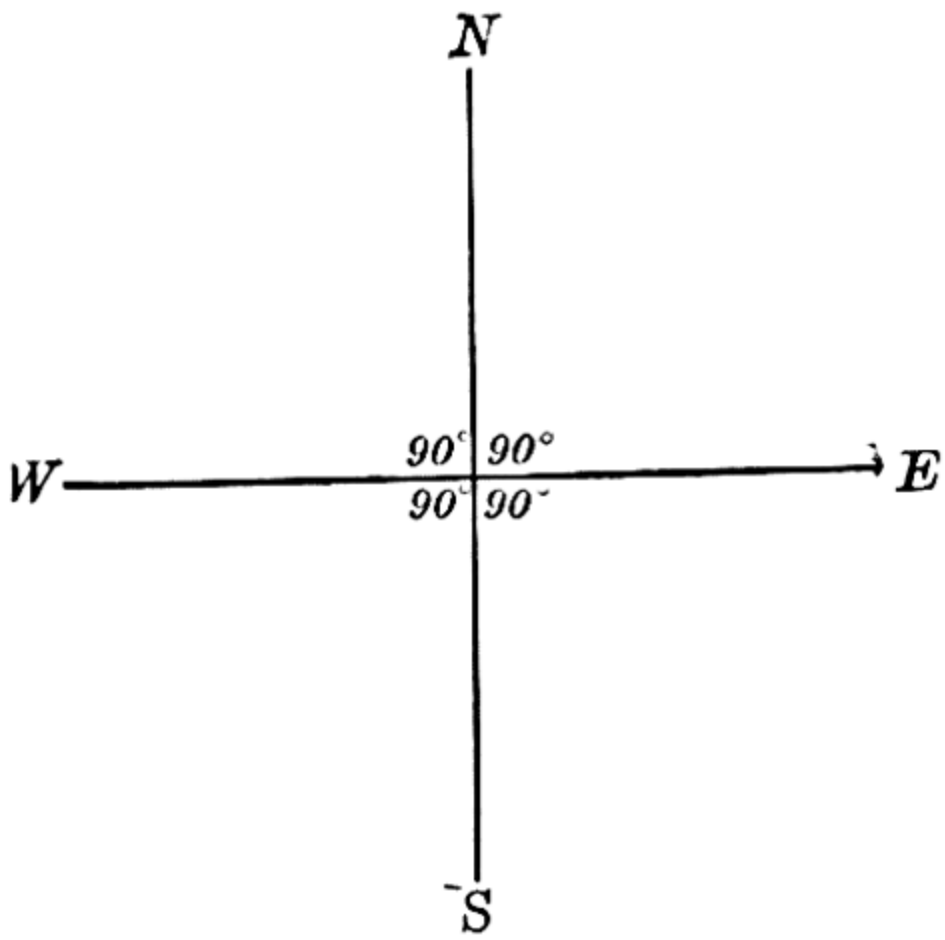


FIG. 25

47. *Perpendicular lines* (⊥) are lines that form right angles with each other. The East-West lines and the North-South lines are perpendicular to each other.

48. *Parallel lines* (∥) are lines that do not meet, however far they are extended. (See lines in Fig. 28.)

EXERCISES

1. Measure each angle in Fig. 26 to determine whether

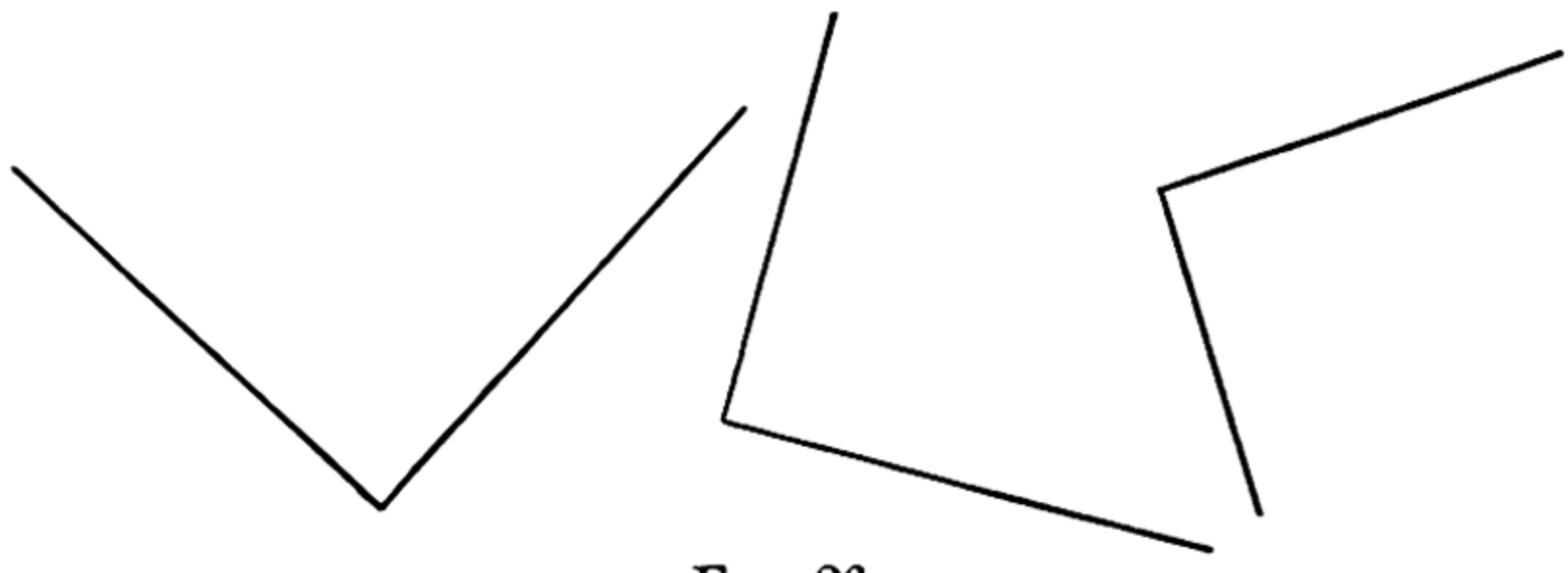


FIG. 26

it is a right angle; that is, whether its sides are *perpendicular* to each other.

2. Place lines in different positions, as in Fig. 27, and draw a perpendicular to each of them by means of a protractor.

FIG. 27

3. Determine by measuring with a protractor the angles in Fig. 28. Do you think that line l_1 will ever meet line l_2, however far each line is extended? What word describes the relative positions of these lines? (See §48.)

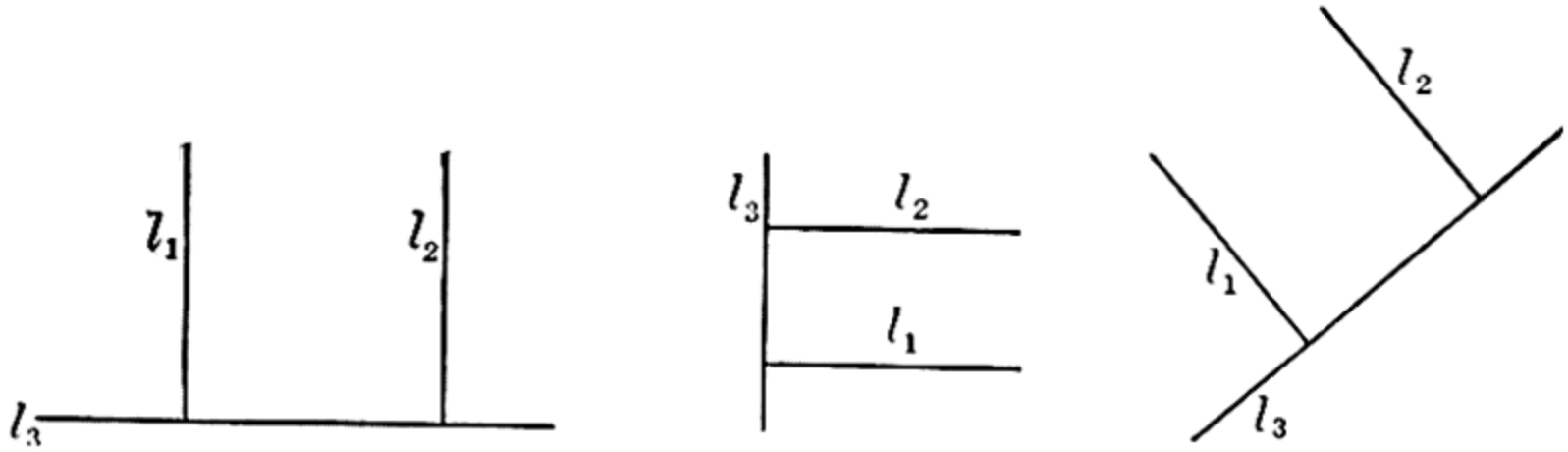
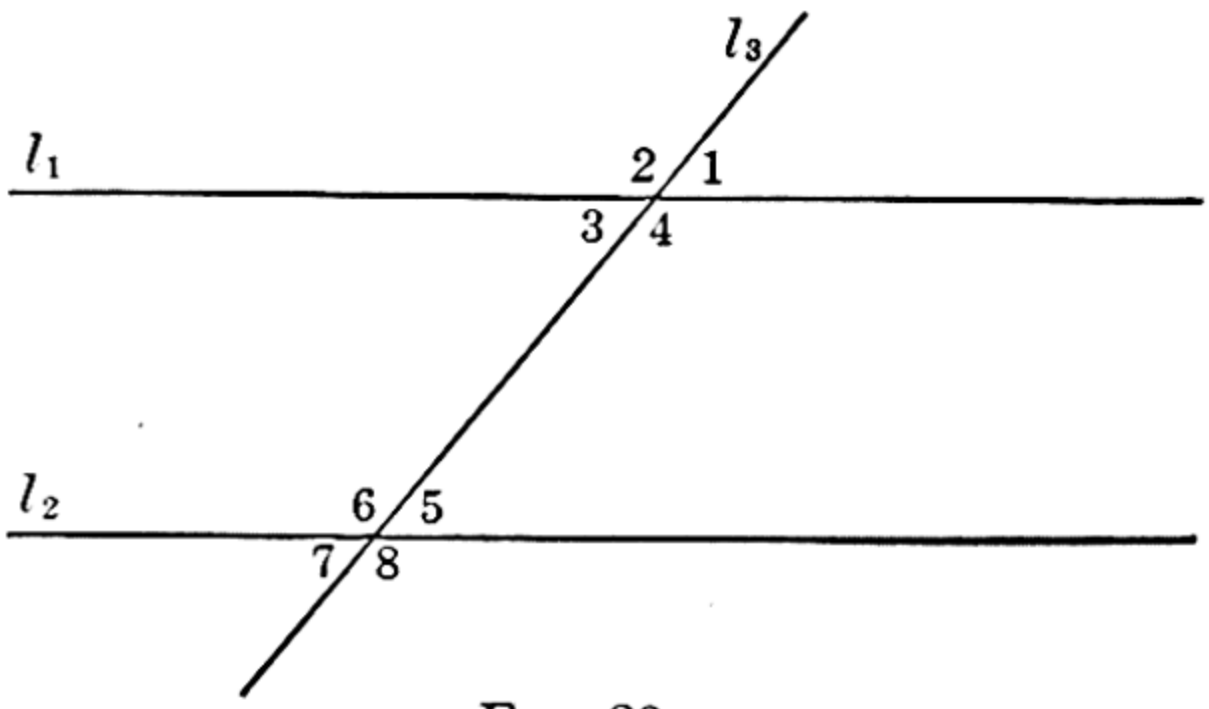


FIG. 28

4. Draw a horizontal line; then erect perpendiculars to this line at two different points. What do you note about these perpendiculars, no matter how long you make them? What name do you give to such lines? Using your protractor draw a perpendicular to one of the latter lines. What relationship does it bear to the first line drawn? Now draw a perpendicular to the other vertical line. What relationship does it bear to the other lines? How many sets of parallel lines have you drawn?

5. Measure each angle in Fig. 29 and write its value in the proper space:



$\angle 1 = ?$
$\angle 2 = ?$
$\angle 3 = ?$
$\angle 4 = ?$
$\angle 5 = ?$
$\angle 6 = ?$
$\angle 7 = ?$
$\angle 8 = ?$

FIG. 29

Which angles seem to be equal? Find all the pairs of angles whose sum is 180°.

Two angles whose sum is 180° are called ***supplementary angles.*** Either angle is called the ***supplement*** of the other angle.

6. Draw two parallel lines and a third line across them. Name the lines and number the angles as in Fig. 29. What angles are equal? What angles are supplementary?

7. How many degrees are there in the supplement of 40°? 100°? 65°? 42°? 137°? 149°? 161°? 107°? (Subtract each angle from 180°.)

8. How many degrees and minutes are there in the supplement of 30° 20′? 138° 15′? 64° 17′? 84° 42′? 122° 47′? 99° 39′? 107° 43′? 137° 14′? 146° 41′? 157° 57′? 175° 55′?

SPECIMEN SOLUTION

180°	or	179° 60′
30° 20′	or	30° 20′
		149° 40′

9. How many degrees, minutes, and seconds are there in the supplement of 4° 15′ 30″? 72° 42′ 20″? 115° 37′ 13″? 12° 44′ 45″? 153° 17′ 35″?

SPECIMEN SOLUTION

$$\begin{array}{rl} 180^\circ & \text{or } 179^\circ\ 59'\ 60'' \\ \underline{\ 4^\circ\ 15'\ 30''} & \text{or } \underline{\ \ 4^\circ\ 15'\ 30''} \\ & \ \ 175^\circ\ 44'\ 30'' \end{array}$$

10. An *acute angle* is an angle less than 90°. Write a list of the acute angles found in Exercises 7, 8, and 9.

11. An *obtuse angle* is an angle greater than 90° but less than 180°. Write a list of the obtuse angles found in Exercises 7, 8, and 9.

12. What kind of an angle, acute or obtuse, do the hands of a clock make at 2:30? At 4:00? At 3:00? At 8:50? At 6:25? At 7:05? At 10:40? At 1:10? At 6:00? At 9:00? At 8:15? At 5:25?

13. Draw three acute angles of different sizes. Measure them; find their sum.

14. Draw three obtuse angles of different sizes. Measure them; find their sum.

15. Draw one acute angle and one obtuse angle. Measure them. Find the ratio of the acute angle to the obtuse angle. What per cent of the obtuse angle is the acute angle?

16. Draw two acute angles of different sizes. Measure them. Find the ratio of the larger angle to the smaller.

17. Draw an acute angle. Measure it. Find its ratio to a right angle. What per cent of a right angle is it?

18. Draw an obtuse angle. Measure it. Find its ratio to a right angle. Reduce this ratio to a per cent.

19. Draw two parallel lines and a third line across them. Measure two of the supplementary angles. Find the ratio of the smaller to the larger.

20. Try to draw supplementary angles by guess so that one shall be two thirds of the other. Measure and find their ratio, to check your guess; make two more trials.

21. Find by computation how many degrees in each of two supplementary angles such that one is two thirds of the other.

SOLUTION. Let n = number of degrees in the larger angle.

Then $\frac{2}{3}n$ = number of degrees in the smaller angle.

Since the sum of the angles is 180°, we have

$$n + \tfrac{2}{3}n = 180.$$

Add the two parts of the first member:

$$\tfrac{5}{3}n = 180.$$

Multiply both members by 3:

$$5n = 540.$$

Divide both members by 5:

$$n = 108.$$

and $\frac{2}{3}n = 72.$

CHECK. $\frac{72}{108} = \frac{2}{3}$; $72° + 108° = 180°$.

Ans. 108° and 72°.

22. One of two supplementary angles is 4 times the other. Find by computation the number of degrees in each and draw the angles.

23. How many degrees in the angle that is one half of its supplement?

24. Two supplementary angles are equal. How many degrees in each?

25. Find the angle that is 9 times its supplement. Draw the angle and its supplement.

26. Suppose you have a given angle or have drawn an angle as in Exercise 25. What is the easiest and most accurate way to draw its supplement?

27. What is the supplement of a right angle? Can two acute angles be supplementary? Can two obtuse angles be supplementary?

II. Plane Figures

49. A ***plane figure*** is a figure which has all of its lines and points lying on a flat surface, or plane.

50. A ***square*** is a plane figure inclosed by four lines of equal length drawn at right angles to each other.

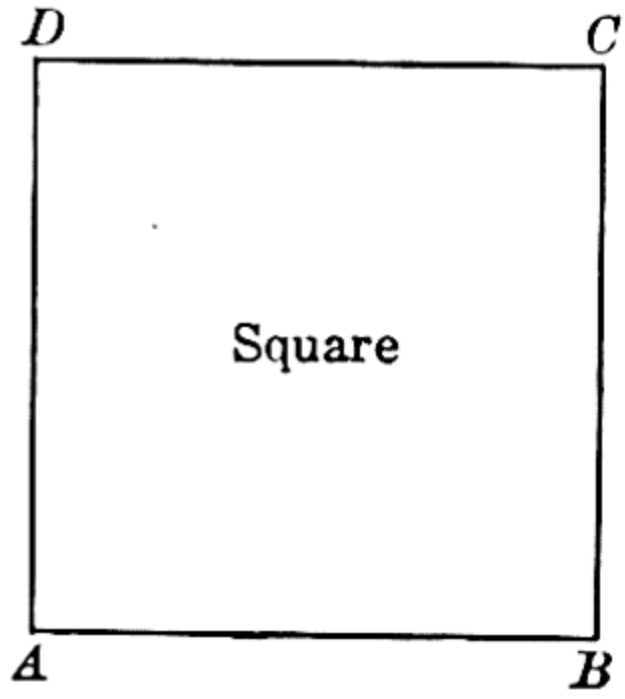


Fig. 30

In the square $ABCD$, $AB = BC = CD = DA$. Angles A, B, C, and D are right angles. AB, BC, CD, and DA are called the *ides* of the square. Points A, B, C, and D are called the ***vertices*** (*plural of* vertex).

EXERCISES

1. Measure the side of the square $ABCD$ to the nearest tenth of an inch. What is the sum of the four sides?

Note. The sum of the sides is called the ***perimeter*** of the square.

2. Draw a square having each side equal to 1.3″. What is its perimeter?

Note. The symbol (′) is commonly used to express feet, and the symbol (″) to express inches.

3. Draw a square whose side is 1.7″. What is its perimeter? Is there a short way of finding the perimeter?

4. Draw a square whose side is $2\frac{1}{8}''$. What is its perimeter?

5. Draw a square whose perimeter is 6.4″. How do you find the length of one side?

6. Draw a square whose perimeter is 5.2″.

7. Draw to the scale of 10 to 1 a square having each side 15″ long. This means that a line 1″ long is drawn to represent a line 10″ long. How long should the line be to represent a 15″ line? How do you divide a number by 10? (See Fig. 31.)

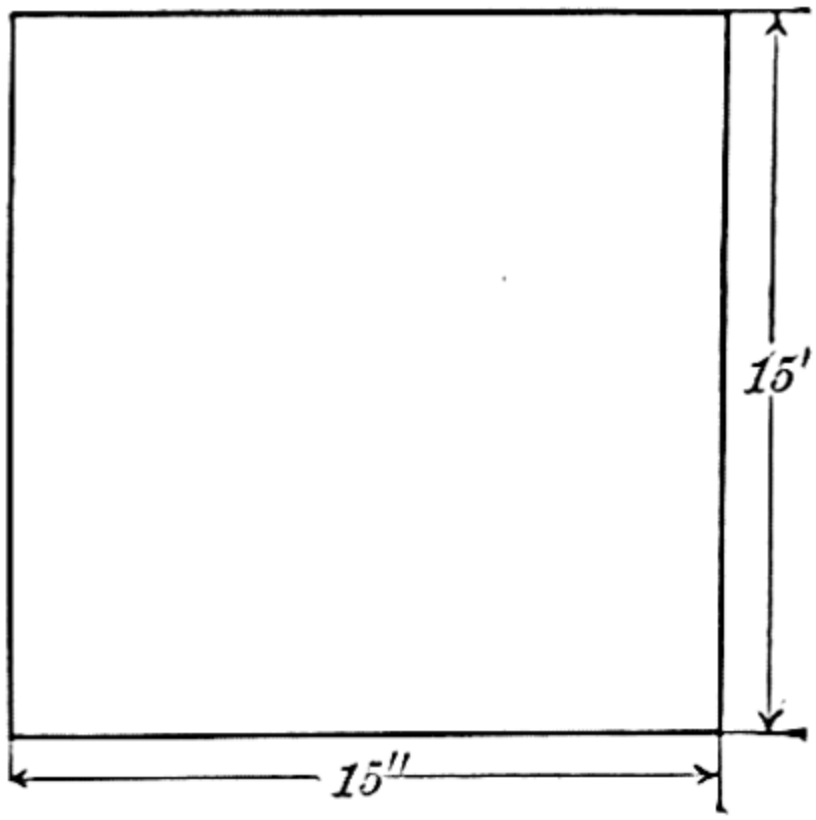


FIG. 31

8. Draw to the scale of 10 to 1 a square having each side equal to 17″. In your figure 17″ should be represented by 1.7″. Why?

9. Draw to the scale of 10 to 1 a square whose perimeter is 48″.

10. Draw to the scale of 20 to 1 a square each side of which is 30″.

11. Draw to the scale of 10′ to 1″ a square each side of which is 18′.

12. A square room is 17′ on each side. Draw a plan of it to the scale of 10′ to 1″.

13. Draw to the scale of 5′ to 1″ a square each side of which is 12′. Here 1″ represents 5′; hence 2.4″ will represent 12′ (since 12 divided by 5 equals 2.4).

14. A square lot of land is 120′ on each side. Draw a plan of the lot to the scale of 100′ to 1″. How do you divide a number by 100?

15. The distance around a square block is 440′. Draw a plan of the block to the scale of 100′ to 1″.

16. Draw to the scale of 50′ to 1″ the plan of a baseball diamond. A baseball diamond is a square 90 feet long. Mark the vertices 1, 2, 3, and 4 for bases, 4 being "Home." Measure the distance on your plan from second base to "Home" to the nearest tenth of an inch. About how many feet does this measurement represent?

51. A *rectangle* is a plane figure inclosed by two pairs of parallel lines at right angles to each other.

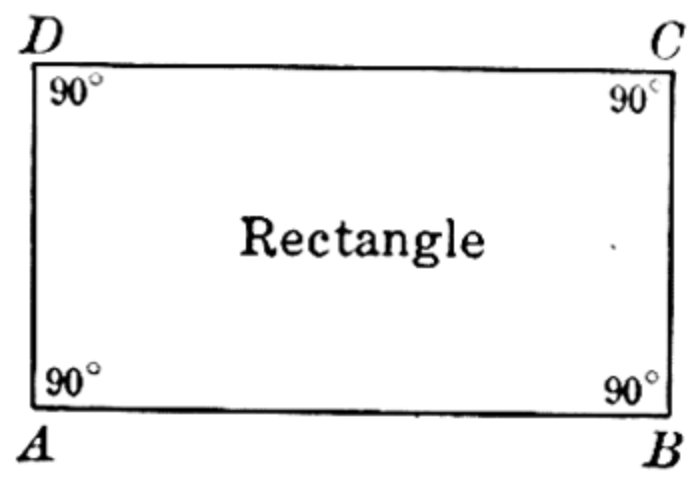


FIG. 32

In the rectangle $ABCD$ the side AB is called the *base*, and the side AD, which is perpendicular to the base, is called the *height* (or *altitude*) of the rectangle.

A square is a special kind of rectangle.

EXERCISES

1. Measure the sides of the rectangle $ABCD$ to the nearest tenth of an inch. (*a*) What is its perimeter? (*b*) What is the ratio of the base to the height?

2. Draw a rectangle whose base is 3.2″ and height 1.4″, using ruler and protractor. (*a*) What is its perimeter? (*b*) Can you find the perimeter in three different ways? (*c*) Which is the shortest way to find the perimeter of a rectangle?

3. Draw a rectangle whose base is $2\frac{1}{4}$″ and whose height is $1\frac{1}{8}$″. (*a*) What is its perimeter? (*b*) What is the ratio of the base to the height? (*c*) What per cent is the base of the height?

4. Draw a rectangle whose base is $2\frac{1}{4}''$ and whose height is $1\frac{7}{8}''$. (*a*) What is its perimeter? (*b*) What is the ratio of the base to the height? (*c*) Express this ratio as a per cent.

5. Draw a rectangle whose base is $3\frac{1}{2}''$ and whose height is $2\frac{5}{8}''$. (*a*) What is its perimeter? (*b*) What is the ratio of its base to its height? (*c*) What per cent of the base is the height?

6. Draw to the scale of 10 to 1 a rectangle whose base is 30″ and whose height is 20″. What is the perimeter of the figure?

7. Draw to the scale of 10′ to 1″ a rectangle whose base is 32′ and whose height is 24′. (*a*) What is the perimeter of the figure? (*b*) What is the ratio of its height to its base? (*c*) What is the ratio of its base to its height? (*d*) What per cent of the height is the base?

8. The base of a rectangle is 2.1″, and its perimeter is 6.8″. Find its height. Draw the rectangle.

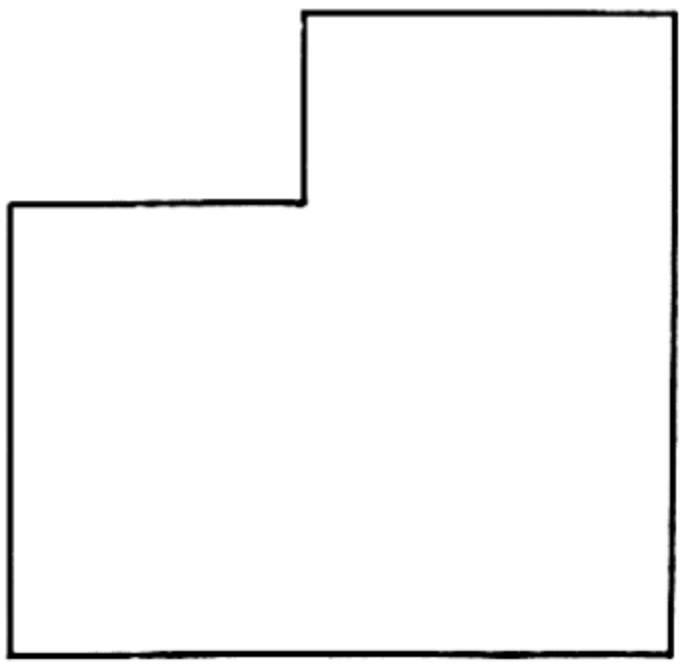


FIG. 33

9. Draw to the scale of 20′ to 1″ the plan of a rectangular lot of land 60′ long and 45′ wide.

10. Draw to the scale of 100′ to 1″ the plan of a rectangular garden plot 225′ long and 180′ wide.

11. Draw a rectangle $1\frac{3}{4}''$ long and 1.4″ wide. (*a*) What is its perimeter? (*b*) What is the ratio of its length to its width?

12. In the diagram of Fig. 33, the scale is 100′ to 1″, and all the angles are right angles. Measure the sides to the nearest tenth of an inch. Find its perimeter.

13. Measure the length and width of your classroom to the nearest tenth of a foot. Draw a plan of it to the scale of 10′ to 1″.

14. If the school yard is rectangular, measure the length and width to the nearest foot. Draw a plan of it to the scale of 100′ to 1″.

52. A ***parallelogram*** is a plane figure inclosed by two pairs of parallel lines.

In the parallelogram $ABCD$ (Fig. 34) AB is called the ***base***, and h, perpendicular to the base, is called the

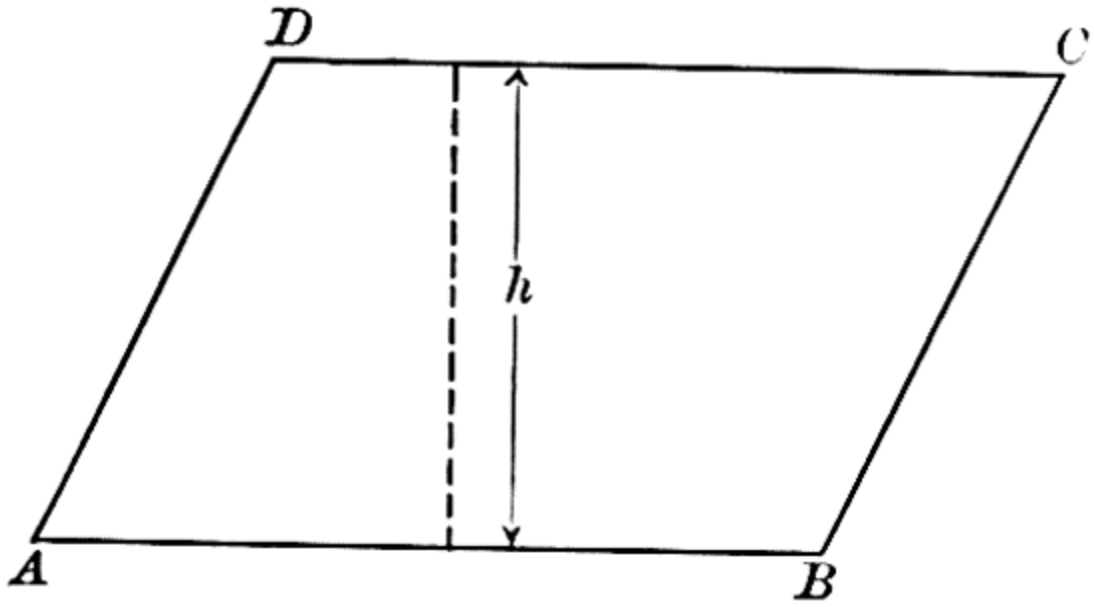


FIG. 34

height (or ***altitude***). In this parallelogram $ABCD$, what kind of angles are the angles at B and D? At A and C?

If a parallelogram were so drawn that all the four angles were equal, what special name would you give the parallelogram?

EXERCISES

1. Measure the four sides of the parallelogram $ABCD$ (Fig. 34) to the nearest tenth of an inch.

(*a*) How do the sides AB and CD compare in length? AD and BC?

(*b*) What is the ratio of BC to AB?

(*c*) Measure its height and find the ratio of the height to the base.

(*d*) What is the perimeter?

(*e*) Using your protractor, compare the opposite angles A and C; also $\angle B$ with $\angle D$.

(*f*) Add the number of degrees in the four angles.

(*g*) $\angle A + \angle B = ?$ $\angle B + \angle C = ?$

(*h*) $\angle A$ and $\angle B$ are supplementary. Why?

2. Draw another parallelogram (not a rectangle). Find the results as required in Exercise 1.

3. By referring to the results obtained in Exercises 1 and 2, complete the following statements:

(*a*) The opposite sides of a parallelogram are (?)

(*b*) The opposite angles of a parallelogram are (?)

(*c*) The two angles at the base of a parallelogram are (?)

(*d*) The sum of the four angles of a parallelogram is equal to (?)

4. The base of a parallelogram is $2\frac{1}{2}''$ long. One angle at the base is 50°.

(*a*) How large is the other angle at the base?

(*b*) Another side of the parallelogram is $1\frac{3}{8}''$ long. Have you enough parts given so that you can draw the whole parallelogram?

(*c*) With your ruler and protractor draw it, if possible.

5. Two of the sides of a parallelogram are 3.2″ and 1.7″; these sides include an angle of 45°. How many degrees in the angle at the other end of the 3.2-inch side? Using your ruler and protractor, draw the parallelogram.

6. Two of the sides of a parallelogram are $2\frac{3}{8}''$ and $1\frac{7}{8}''$; these sides include an angle of 55°. Draw the parallelogram.

7. Two of the sides of a parallelogram are 24″ and 18″; these sides include an angle of 40°. Draw the parallelogram to the scale of 10 to 1. The sides of the parallelogram drawn to scale will be 2.4″ and 1.8″ but the angle will remain 40°. In making drawings to scale the angles will not change because the angles fix the direction of the sides.

8. Two of the sides of a parallelogram are 40″ and 30″; these sides include an angle of 65°. Draw the parallelogram to the scale of 20 to 1. What will the lengths of the sides be?

9. Two of the sides of a parallelogram are 50′ and 15′; the included angle is 75°. Draw the parallelogram to the scale of 20′ to 1″.

10. Two of the sides of a parallelogram are 120′ and 90′; the included angle is 115°. Draw the parallelogram to the scale of 100′ to 1″.

53. A *triangle* (△) is a plane figure inclosed by three straight lines.

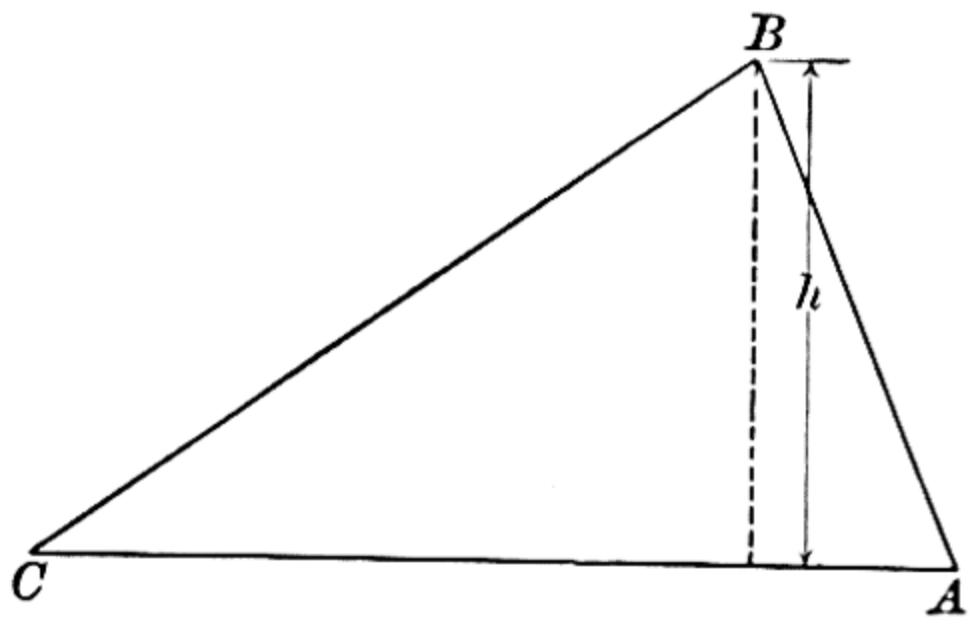


FIG. 35

In the triangle ABC (Fig. 35) AC is called the *base*, and h, perpendicular to the base, is called the *height* (or *altitude*).

EXERCISES

1. With your ruler measure the three sides and the height of triangle ABC (Fig. 35) to the nearest tenth of an inch.

(a) What is the perimeter?

(b) What is the ratio of AB to AC?

(c) What is the ratio of BC to AC?

(d) What is the ratio of the height to the base?

(e) What per cent of the base is the height?

2. With your protractor measure the three angles of the triangle ABC.

(a) What kind of angle is each, right, acute, or obtuse?

(b) What is the ratio of $\angle A$ to $\angle C$?

SOLUTION. By measurement,

$$\angle A = 68°, \ \angle C = 35°;$$

$$\text{then } \frac{\angle A}{\angle C} = \frac{68}{35}$$

Ans. The ratio of $\angle A$ to $\angle C = 68$ to 35.

(c) Find the sum of the three angles.

3. With your ruler draw a triangle having all its sides of different lengths. Draw the height.

(a) Measure the three sides and the height to the nearest tenth of an inch. What is the perimeter of the triangle to the nearest tenth of an inch?

(b) Measure the three angles. The sum of the three angles = (?)°

4. Two sides of a triangle are 2.4″ and 1.8″ long; these sides include an angle of 50°. Draw the triangle. Measure the other two angles. Find the sum of the three angles.

5. Two sides of a triangle are $2\frac{1}{4}''$ and $\frac{7}{8}''$; these sides include an angle of 42°. Draw the triangle.

(*a*) Measure the third side. Find the perimeter of the triangle.

(*b*) Measure the other two angles. Find the sum of the three angles.

6. Two sides of a triangle are 23″ and 17″; these sides include an angle of 68°. Draw the triangle to the scale of 10 to 1.

(*a*) Find the perimeter of the triangle.

(*b*) Measure the other two angles. Find the sum of the three angles.

7. A grass plot is triangular. Two of the sides are each 120′ in length; they include an angle of 65°. Draw the plan of the plot to the scale of 100′ to 1″.

(*a*) Find its perimeter.

(*b*) Measure the other two angles. Find the sum of the three angles.

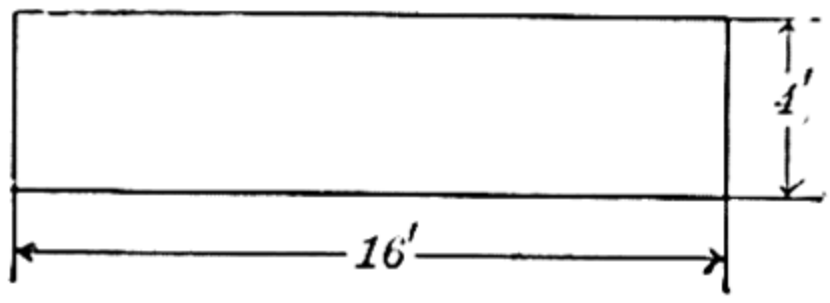


FIG. 36

8. This plan, Fig. 36, is drawn to the scale of 10′ to 1″. What fractional part of the base is the height? What per cent of it?

9. The height of a rectangle is 5′ and the base is 25′. What fractional part of the base is the height? What per cent of it?

III. Representation of Number Data by Pictures, by Line Graphs, and by Tables

54. Pictures are often used in newspapers, magazines, and reports by the Government to show the relation between numbers since the picture gives the facts in an easy and convincing way. For example, Figs. 37 and 38 are taken from the *Yearbook* of the United States Department of Agriculture, where you will find many interesting graphical representations of number data. If copies of this book are available, it will be helpful to have some of the most striking illustrations reproduced on the blackboard by members of the class who draw well.

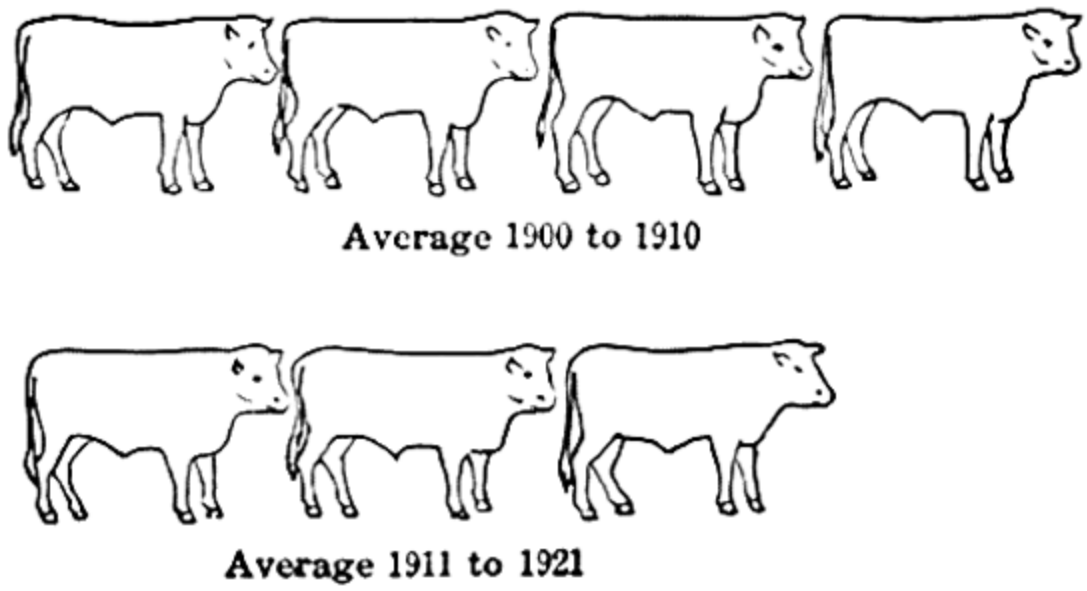


Fig. 37

1. What was the average number of beef cattle in the United States for every 10 people from 1900 to 1910? From 1911 to 1921? Was there an increase or decrease per person?

What is the ratio of the number represented in the bottom row to the top row? Express this ratio as a per cent.

2. How many bushels of wheat were exported during each of the ten-year periods shown in the picture on this page (Fig. 38)?

During which periods were the annual exports about equal in amount?

How many times as many bushels were exported in the last period (1911 to 1920) as in the first period (1871 to 1880)? As in the fourth period (1901 to 1910)?

Can you think of any reasons why the number of bushels of wheat exported in 1911-1920 should be so much greater than during any other period? Would you expect that the average would be as large in the next ten years, 1921–1930?

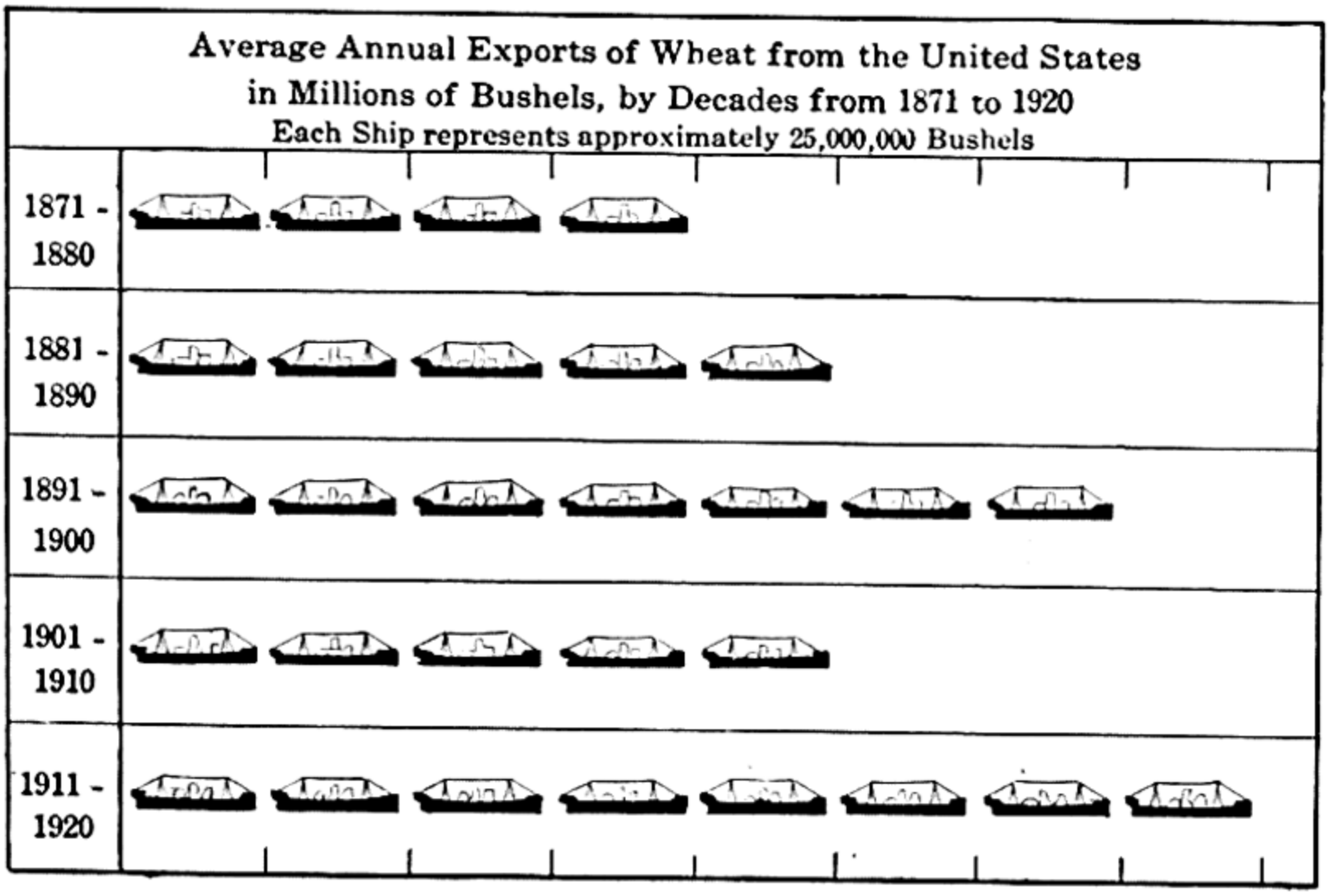


FIG. 38

55. Line measurements are used frequently to represent the amounts of other quantities. The following exercises, several of which are explained in detail, make clear how this is done.

EXERCISES

Dams Built by U. S. Government for Irrigation Purposes		
		Height
(1) Roosevelt (Ariz.)	1911	280 ft.
(2) Arrowrock (Idaho)	1915	349 ft.
(3) Pathfinder (Neb.-Wyo.)	1909	218 ft.
(4) Elephant Butte (N. Mex.)	1916	306 ft.
(5) Shoshone (Wyo.)	1910	328 ft.

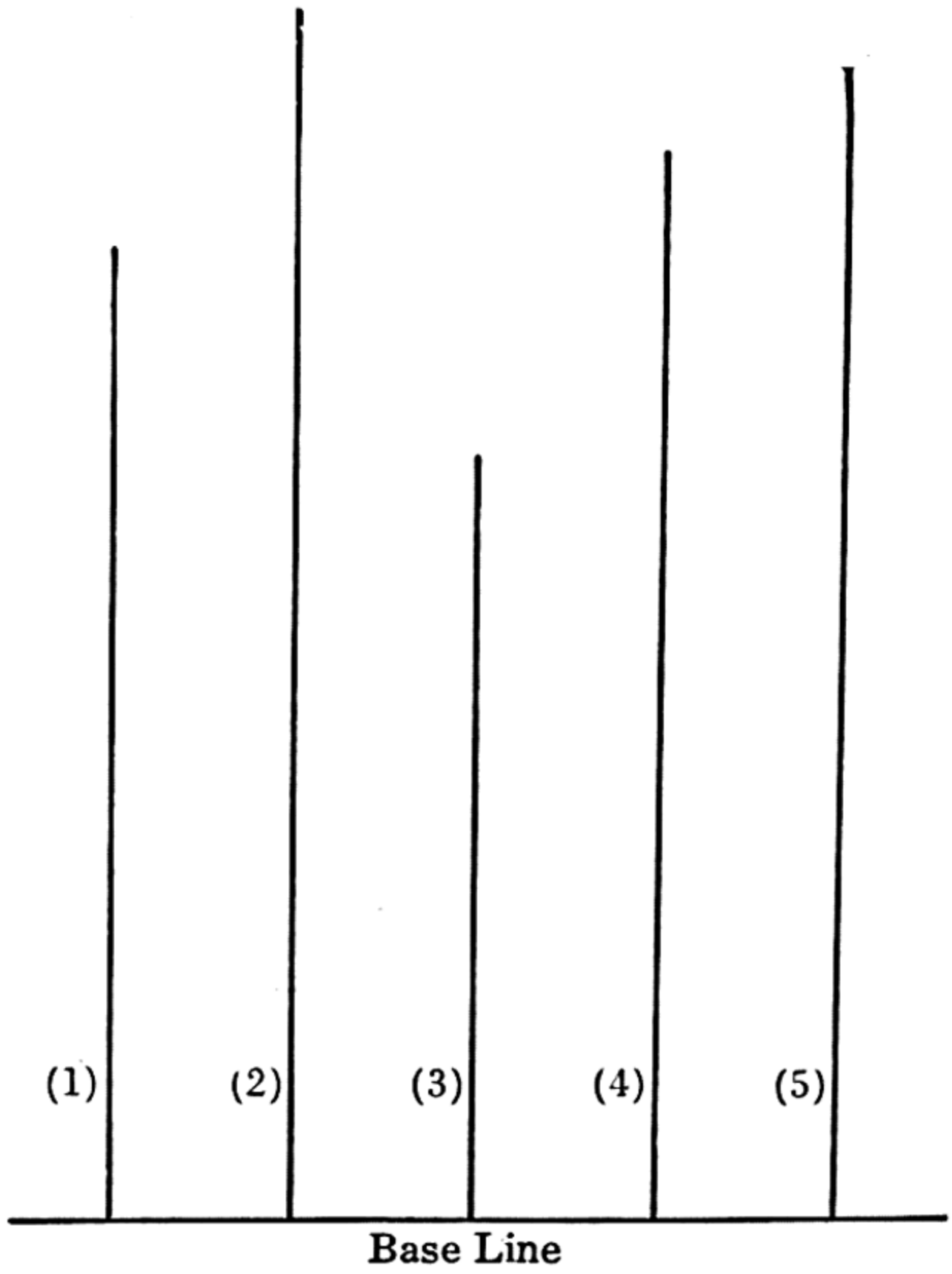


FIG. 39

1. The heights of dams built by the United States Government for irrigation purposes are represented by the *vertical* lines in Fig. 39. One inch represents 100 feet. Measure each line to the nearest tenth of an inch and verify the data given in the table.

2. Figure 40 represents per capita production and consumption of wheat. What does the expression "per capita" mean? When was the per capita production the least? When the greatest?

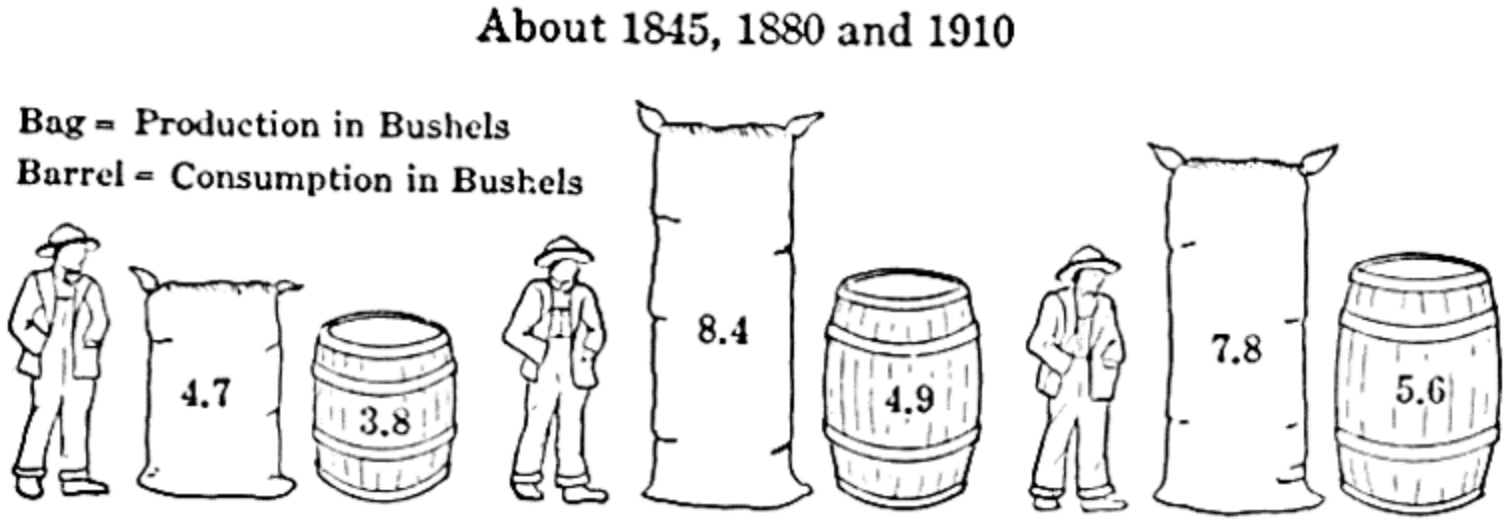


FIG. 40

When was the per capita consumption the greatest? When the least?

From this illustration would you say that the people of the United States will soon consume all the wheat raised within the United States?

In which period was the ratio of the production to the consumption greatest? (To answer this question, compute the value of the following ratios to two decimal places: $\frac{47}{38} = ?$; $\frac{84}{49} = ?$; $\frac{78}{56} = ?$)

Express each of these ratios as a per cent.

3. The greatest length of each of the five Great Lakes of the United States is represented by the horizontal lines in Fig. 41. One inch represents 200 miles. Measure each

line to the nearest tenth of an inch and verify the data in the following table.

LENGTH OF THE FIVE GREAT LAKES OF THE UNITED STATES

	Greatest Length in Miles
(1) Superior..	360
(2) Michigan.	307
(3) Huron....	206
(4) Erie.....	241
(5) Ontario...	193

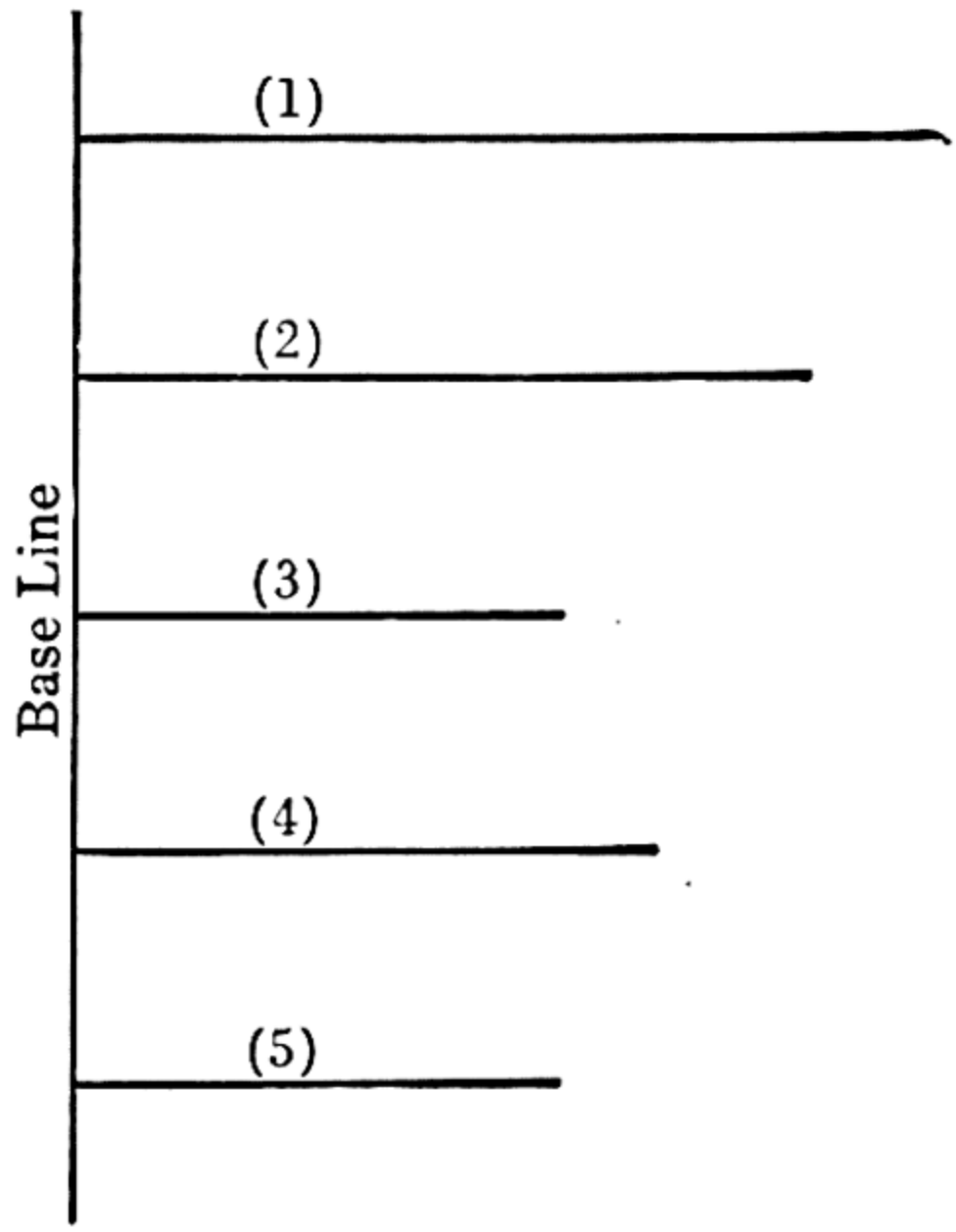


FIG. 41

4. The following is the United States Government table of heights and weights of children from 10 to 14 years of age.

UNITED STATES GOVERNMENT TABLE OF HEIGHTS AND WEIGHTS OF CHILDREN

Age	Boys		Girls	
	Height (Inches)	Weight (Pounds)	Height (Inches)	Weight (Pounds)
10 years.....	51.7	65.3	51.3	62.4
11 years.....	53.3	70.2	53.4	68.8
12 years.....	55.1	76.9	55.9	78.3
13 years.....	57.2	84.8	58.2	88.7
14 years.....	59.9	94.9	59.9	98.4

Using a scale of 10 to 1, represent the heights of boys from 10 to 14 years of age by *vertical-line* graphs. (See Fig. 39 for suggestions as to the way to do this.) Also represent the heights of girls from 10 to 14 years of age by vertical-line graphs.

5. Draw the vertical lines of the line graph for the boys' height in Exercise 44 so they will be the same distance apart. Connect the upper ends by a series of straight lines, making a continuous broken line. The "rise" of this broken line as it proceeds toward the right pictures the rate of growth during the years 10–14.

You recognize this broken line as the kind of graph you have been using to record your progress in certain proficiency tests. [See Chap. II, §17.] If a vertical line is drawn midway between the lines for 11 years and 12 years, its length from the base line to the graph is approximately the average height for 11½ years. The length of the vertical line from *any* point on the base line up to the graph is approximately the average height for the age represented by the location of that point on the base line. Such graphs we will call *continuous graphs.*

6. On the same piece of cross-ruled paper draw the two continuous graphs showing the change in the height of the boys and the girls according to the table. Which grows the faster from 11 to 13 years? Which from 13 to 14?

7. Give a good reason why the data of Exercises 1 and 3 can not be illustrated by continuous graphs.

8. Using a scale of 10 to 1, represent the heights of the boys in your class by vertical-line graphs. (If the heights of the boys are not known, probably your teacher will direct you in obtaining them by measurement.)

9. Using a scale of 10 to 1, represent the heights of the girls in your class by vertical-line graphs.

10. Using a scale of 10 to 1, represent the weights of boys from 10 to 14 years of age by horizontal-line graphs.

11. Using a scale of 10 to 1, represent the weights of girls from 10 to 14 years of age by horizontal-line graphs. Could the data of Exercises 9 and 10 be pictured by continuous graphs?

12. The average number of people per square mile in certain states in 1920 is given in the following table:

State	Population per Square Mile	State	Population per Square Mile
Rhode Island	566.4	Illinois	115.7
Massachusetts	479.2	Delaware	113.5
New Jersey	420.0	Indiana	81.3
Connecticut	286.4	Michigan	63.8
New York	217.9	West Virginia	60.9
Pennsylvania	194.5	Kentucky	60.1
Maryland	145.8	Virginia	57.4
Ohio	141.4	Tennessee	56.1

Represent by horizontal-line graphs the average number of people per square mile in each of the states given above. Let one inch represent 100 people. (If your state is not in the above list, consult your geography for these data.)

13. Represent the heights of the following buildings in New York City by vertical-line graphs, letting 1 inch represent 200 feet.

Woolworth	792 feet
Metropolitan	700 feet
Singer	612 feet
Equitable	486 feet
Times	410 feet
Flatiron (Fuller)	286 feet

14. Represent the length of the following rivers by line graphs, letting 1 inch represent 1000 miles.

Amazon	3,800 miles	Niger	2,900 miles
Amur	2,600 miles	Nile	3,766 miles
Congo	3,000 miles	Rhine	700 miles
Danube	1,725 miles	Rio Grande	1,800 miles
Hwang	2,600 miles	St. Lawrence	2,150 miles
La Plata	2,300 miles	Volga	2,300 miles
Mackenzie	2,525 miles	Yangtze	3,400 miles
Mississippi-Missouri	4,200 miles	Yenisei	3,300 miles

15. Miles of electric railway tracks (1915) were distributed as represented by the following line graphs, where 1 inch represents 5,000 miles.

NEW ENGLAND STATES — $1.22 \times 5000 = 6000$ *miles*

EASTERN STATES

CENTRAL STATES

SOUTHERN STATES

WESTERN STATES

FIG. 42

Measure each line in Fig. 42 to the nearest hundredth of an inch, if possible. (To get the nearest hundredth, think of the tenth of the inch as divided into ten equal divisions. These divisions are very small, and you will have to estimate the number of them.) By multiplying each number (obtained by the measurement of each line) by 5000, find the number of miles of electric railway tracks in each section of the United States.

These numbers are not the exact numbers, but they are as near the exact numbers as you can get from the measurements made. Such numbers are often called "round" numbers (and sometimes "approximate" numbers).

16. The numbers of miles of railway tracks (1920) for certain states were as follows:

Texas	16,125
Illinois	12,188
Pennsylvania	11,551
Iowa	9,808
New York	8,390
Massachusetts	2,106

Make line graphs for these, letting 1 inch represent 5000 miles.

Texas: $\frac{16,125}{5,000} = 3.23$ to the nearest hundredth.

To get the nearest hundredth find out, by dividing, what the figure in the *thousandths'* place is. If that figure is 5 or more, the *nearest* hundredth is 1 higher than you get by dividing; if that figure in the thousandths' place is less than 5, the figure in the hundredths' place is not changed.

17. Represent the lengths of the following bridges in New York City by horizontal-line graphs, letting 1 inch represent 500 feet.

Brooklyn	6,016 feet
Manhattan	6,855 feet
Williamsburg	7,308 feet
Queensboro	7,449 feet
Hell Gate Arch	18,000 feet

18. Represent the heights of the following mountains by vertical-line graphs, letting 1 inch represent 5000 feet.

Mt. McKinley	20,300 feet	Mt. Everest	29,002 feet
Mont Blanc	15,782 feet	Mt. Aconcagua	23,080 feet

19. Represent the number of people speaking the following languages (1910) by horizontal-line graphs, letting 1 inch represent 50,000,000 people.

English	160,000,000
German	130,000,000
French	70,000,000
Russian	100,000,000
Italian	50,000,000

20. Represent by horizontal-line graphs the relative sizes of the following islands, using different scales for the two groups.

Group I		Group II	
Bahamas	5,400 sq. mi.	Borneo	284,000 sq. mi.
Corsica	3,400 sq. mi.	Greenland	827,300 sq. mi.
Crete	2,900 sq. mi.	Japan	160,000 sq. mi.
Hawaiian	6,450 sq. mi.	Madagascar	227,000 sq. mi.
Jamaica	4,200 sq. mi.	New Guinea	330,000 sq. mi.
Long Is.	1,376 sq. mi.	Philippines	115,026 sq. mi.

QUESTIONS FOR REVIEW

1. Why can a carpenter tell when a board is not straight by looking along the edge? Can a carpenter make a crooked board straight? A rough board smooth? Explain.

2. What principle does a farmer use when he "sights" along two fence posts to see whether the third one, when set, will be in a straight line with the other two?

3. An insurance inspector has to estimate the floor areas of all the rooms in any building which he inspects. Can you think out a method by which he can determine quite accurately the length of his step so that he can always use the "pacing" method in making these measurements?

4. A certain surveyor says that he can find the angle which two lines that he is running in surveying make with each other by adjusting the hands on his watch. Can you explain how he does this?

5. Can you find out how to use a watch at 12 noon to determine the points of the compass? When would you use this method?

6. Tell which of the following statements are true and which are false:

(*a*) A rectangle is a square.
(*b*) A square is a rectangle.
(*c*) A rectangle is a parallelogram.
(*d*) A parallelogram is a rectangle.

7. Complete the following statements:

(*a*) The sum of the angles of a parallelogram is ———
(*b*) The sum of the angles of a triangle is ———
(*c*) The angles of a rectangle are ———
(*d*) The opposite angles of a parallelogram are ———

8. To find the perimeter of any parallelogram, you ——— (give three ways).

9. A farmer asked his son to figure out the number of hills of corn that he could plant in a field 20 rods long and 8 rods wide. The hills were to be in rows 36 inches apart and there was to be 24 inches between hills in each row. Solve this problem.

CHAPTER VI

MENSURATION

I. Areas

56. A ***square inch*** is a square having each side one inch long. A ***square foot*** is a square having each side one foot long. A ***square yard*** is a square having each side one yard long.

The surface inclosed by any figure may be expressed as a certain number of square inches or square feet or square yards, etc.

The square inch, the square foot, the square yard are known as the ***units of square measure.***

Table of Square Measure

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

57. Rectangles. The process involved in finding areas of rectangles is illustrated by the following examples. The definition of a rectangle and the method of construction are given on page 113.

Example 1. Draw a rectangle 3″ long and 2″ wide. Divide it by lines as in Fig. 43 on page 132.

The rectangle here is made up of 6 squares, each an inch long and an inch wide, or six *square inches*. There are two rows of three squares each

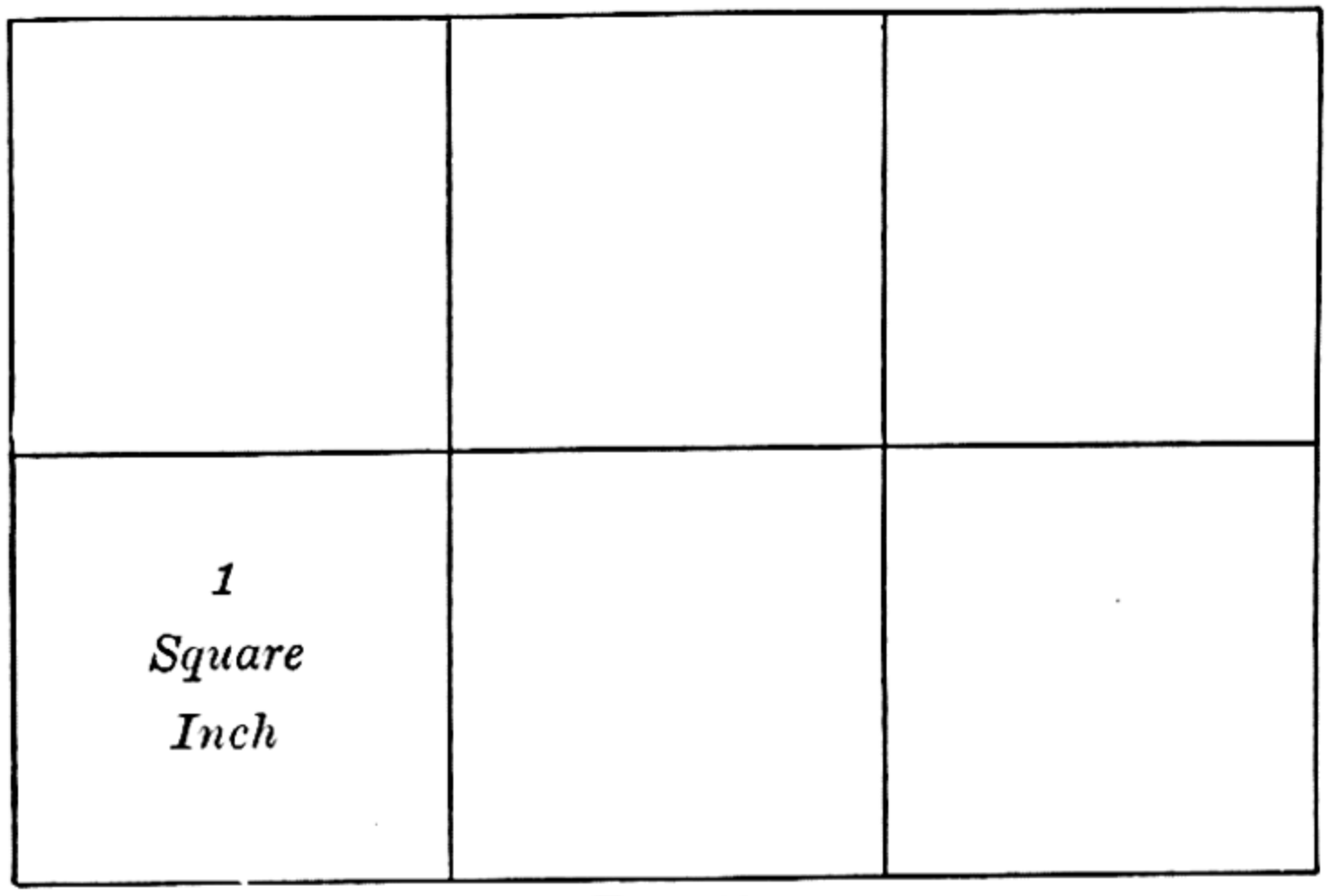


FIG. 43

The number of squares thus formed gives the ***area*** of the rectangle; for example, the area of the rectangle drawn above is 6 square inches.

If we let A stand for the number of square units in the area of a rectangle, b stand for the number of linear units in the base, and h stand for the number of linear units in the height (or altitude), then

A (the number of square units) $= (b \times h)$ square units. When the multiplication of letters is indicated, the sign $\times$ is generally omitted.

The expression $A = bh$ is the ***formula for the area of a rectangle.*** A *formula* is a brief way of expressing a rule.

EXAMPLE 2. The base of a rectangle is 4″ and its height is 3″. Find its area.

SOLUTION. The formula for the area of a rectangle, when the base and height are given in inches, is

A (the number of square inches) = bh square inches.
Substitute 4 for b and 3 for h:

$A = (4 \times 3)$ square inches, or
$A = 12$ square inches.

Ans. $A = 12$ sq. in.

EXAMPLE 3. The base of a rectangle is 7′ and its height is 6′. Find its area.

SOLUTION. A (the number of square feet) = bh square feet.

Substitute 7 for b and 6 for h:

$A = (7 \times 6)$ square feet, or
$A = 42$ square feet.

Ans. $A = 42$ sq. ft.

EXAMPLE 4. The two sides of a rectangle are 1′ 6″ and 1′ 3″. Find its area.

SOLUTION. The base = $1\frac{1}{2}'$ and the height = $1\frac{1}{4}'$.

A (the number of square feet) = bh square feet.
$A = (1\frac{1}{2} \times 1\frac{1}{4})$ square feet,

(*Estimate:* 2×1, or 2, sq. ft.)

$A = (\frac{3}{2} \times \frac{5}{4})$ square feet, or
$A = \frac{15}{8}$ square feet, or
$A = 1\frac{7}{8}$ square feet.

Ans. $A = 1\frac{7}{8}$ sq. ft.

EXAMPLE 5. The base of a rectangle is 1′ 2″ and the height is 8″. Find its area in square inches.

SOLUTION.

A (the number of square inches) = bh square inches.
$A = (14 \times 8)$ square inches, or
$A = 112$ square inches.

Ans. $A = 112$ sq. in.

EXAMPLE 6. The length of a rectangle is 3.4″ and the width is 2.5″. Find its area.

SOLUTION.

A (the number of square inches) = bh square inches.

(*Estimate:* 3×3, or 9, sq. in.)

$A = (3.4 \times 2.5)$ square inches, or

$A = 8.5$ square inches.

Ans. $A = 8.5$ sq. in.

EXERCISES

Copy and fill in this table for areas of rectangles.

FORMULA: $A = bh$

	b	h	ESTIMATE	A
1.	8″	$5\frac{1}{2}$″	(8 × 5) sq. in.	44 sq. in.
2.	$10\frac{3}{4}$″	8″	(11 × 8) sq. in.	
3.	$10\frac{1}{2}$″	$5\frac{1}{2}$″	(10 × 6) sq. in.	
4.	12′	$11\frac{1}{4}$′	? sq. ft.	
5.	$16\frac{2}{3}$′	12′		
6.	$20\frac{1}{2}$′	$18\frac{3}{4}$′		
7.	15 yd.	18 yd.	? sq. yd.	
8.	$23\frac{1}{3}$ yd.	15 yd.		
9.	16 rd.	20 rd.	? sq. rd.	
10.	5.4″	6.5″	? sq. in.	
11.	18.5″	12.4″		
12.	8.6′	9.4′	? sq. ft.	
13.	30.8′	20.5′		
14.	12′ 2″	6′	? sq. ft.	
15.	15′ 9″	4′ 6″	? sq. ft.	
16.	42″	27″	? sq. ft.	
17.	15.8′	4.5′		
18.	24.6′	15.5′		
19.	21.4 yd.	16.5 yd.	? sq. yd.	
20.	32.8 yd.	24.5 yd.		
21.	7′ 8″	106″	? sq. ft.	
22.	3 yd. 2 ft.	12 yd.		
23.	5 yd. 6 in.	12 ft. 6 in.	? sq. ft.	
24.	34.8 ft.	15.4 yd.	? sq. yd.	

25. A room is 13′ by 16′. Draw to the scale of 10′ to 1″ a plan of the room. Find the area of the floor of the room.

26. A lot of land is 65′ by 30′. Draw to the scale of 20′ to 1″ a plan of the lot. Find the area of the lot.

27. Draw to the scale of 100′ to 1″ the plan of a lot of land 180′ by 125′. Find the area of the lot.

28. The four walls of a square room are each three yards high and five yards wide. How many square yards of paper will be required to paper the walls allowing for no waste? How many square yards would be required for the job if an allowance of ten per cent extra were made for waste?

29. Each of the four panes of glass in a window measures 24″ by 15″. How many square inches of glass are there in the window? How many square feet?

30. The two blackboards in a certain classroom are three feet high. One board is twelve feet long and the other is fifteen feet long.

(*a*) How many square feet of blackboard are there in the room?

(*b*) What is the ratio of the area of the smaller board to that of the larger?

(*c*) The smaller board is what per cent of the larger board?

31. Measure the length and width of each of the blackboards in your classroom to the nearest tenth of a foot.

(*a*) How many square feet of board are there in your classroom?

(*b*) If there are two or more boards, what is the ratio of the area of the smallest to that of the largest?

(*c*) What per cent is the smallest of the largest?

For Exercises 31–34, the measurements should be made by several members of the class independently. These measurements should be tabulated and averaged before the areas are computed. Make a table for each as follows:

	LENGTH	WIDTH
1. 2. 3. 4. 5. 6.		
AVERAGE		

32. Measure the length and width of the top of your desk to the nearest tenth of an inch.

(*a*) How many square inches does it contain?

(*b*) How many square feet does it contain?

(*c*) How many square feet of wood are there in the tops of all the pupils' desks in the room?

33. Measure the length and width of the floor of your classroom to the nearest tenth of a foot.

(*a*) How many square feet does it contain?

(*b*) Find the ratio of the blackboard space to the floor space.

34. Measure the length and width of one window pane to the nearest tenth of an inch.

(*a*) Find the area of one pane of glass.

(*b*) Find the total area of all the panes of glass in the windows of your classroom.

(*c*) The window area is what per cent of the floor area?

35. One room is 18′ long and 15′ wide. A second room is 12′ long and 15′ wide.

(*a*) Find the area of each of the floors.

(*b*) The area of the floor of the second room is what fractional part of the area of the floor of the first? What per cent?

(*c*) The first area is how many square feet larger than the second?

(*d*) The difference obtained in (*c*) is what fractional part of the floor area of the first room? What per cent?

(*e*) The difference obtained in (*c*) is what fractional part of the floor area of the second room? What per cent?

36. A certain classroom is 40′ by 30′. The assembly hall is 120′ by 80′.

(*a*) Find the floor area of each.

(*b*) The floor area of the classroom is what fractional part of the floor area of the assembly hall? What per cent?

(*c*) What is the difference in the areas?

(*d*) The difference obtained in (*c*) is what fractional part of the floor area of the assembly hall? What per cent?

(*e*) The difference obtained in (*c*) is what fractional part of the floor area of the classroom? What per cent?

37. On a map drawn to the scale of 200 miles to the inch, what area would be inclosed by a strip 3″ by 2″?

38. On a map drawn to the scale of 50 miles to the inch, what area would be inclosed by a strip $1\frac{1}{2}$″ by 3″?

39. On a map drawn to the scale of 40 miles to the inch, what area would be inclosed by a strip 1.4″ by 2.2″?

40. A lawn 36′ by 30′ is to be sodded. How many pieces of sod will be needed if each piece is 12″ by 15″?

41. Find the cost of a cement sidewalk 40′ long and 4′ 9″ wide at 50 cents a square foot.

42. Find the cost of a cement sidewalk 60′ long and 4′ 6″ wide at 55 cents a square foot.

43. A rectangular lot of land 60′ by 50′ has a walk 5′ wide around it. Draw a diagram, scale 10′ to 1″.

(*a*) What is the area of the lot?

(*b*) What is the area of the walk?

(*c*) The area of the walk is what per cent of the area of the lot?

(*d*) Find the cost of cementing the walk at 50 cents a square foot.

44. The area of a rectangle is 6 sq. in.; its base is 3 inches long. Find its height.

SOLUTION. Place 6 for A and 3 for b in the formula,

$$A = bh.$$

Thus, $$6 = 3\,h.$$

Divide each member by 3:

$$2 = h.$$

CHECK. $$6 = 3 \times 2.$$

Ans. $h = 2''$.

The rectangle in Fig. 43, page 132, contains 6 square inches. The base is 3 inches long; so the rectangle contains 3 columns of squares having 2 squares in each column; hence the height is 2″.

45. The area of a rectangle is $12\frac{1}{2}$ sq. in.; the height is $2\frac{1}{2}''$. Find its base.

SOLUTION. Place $12\frac{1}{2}$ for A and $2\frac{1}{2}$ for h in the formula,

$$A = bh.$$

Thus, $$12\tfrac{1}{2} = b \times 2\tfrac{1}{2}.$$

Divide each member by $2\frac{1}{2}$:

$$5 = b.$$

CHECK. $$12\tfrac{1}{2} = 2\tfrac{1}{2} \times 5.$$

Ans. $b = 5''$.

Following the models in Exercises 44 and 45 copy the following table and find the required values.

FORMULA: $A = bh$

	A		ESTIMATE	EQUATION	?
46.	50 sq. in.	$b = 10''$	$5''$	$50 = 10\,h$	$h = 5''$
47.	90 sq. ft.	$h = 7\frac{1}{2}'$		$90 = 7\frac{1}{2}\,b$	$b = 12'$
48.	150 sq. in.	$b = 15''$			$h = ?$
49.	6.5 sq. in.	$h = 5''$			$b = ?$
50.	$12\frac{1}{2}$ sq. ft.	$b = 2\frac{1}{2}'$			$h = ?$
51.	225 sq. in.	$b = 1'\ 3''$			$h = ?$
52.	18.9 sq. in.	$h = 3.15''$			$b = ?$
53.	$18\frac{3}{8}$ sq. ft.	$h = 3\frac{1}{2}'$			$b = ?$

54. Find the length of a gymnasium floor 45 feet wide that contains 4050 square feet.

55. A football field containing 48,000 square feet is 300 feet long. How wide is it? What is its perimeter?

56. A lot of land contains 2394 square feet and its length is 36 feet. How wide is it? How long must the fence be that incloses it?

57. Out of a piece of cardboard 24″ by 36″, small cardboards are to be cut 18″ long and having a surface measure of 216 square inches. How many of these smaller boards can be cut out of each large board? Draw a diagram.

58. For a room 12 feet by 16 feet find which width of printed linoleum would be most economical, if the choice is to be made from the following:

(*a*) 6 feet wide at \$1.60 per running yard,
(*b*) 9 feet wide at \$2.30 per running yard,
(*c*) 12 feet wide at \$3.00 per running yard.

Find the total cost, and draw a diagram, in each case.

58. Squares. A *square* is a special kind of rectangle. The base and height of the square are equal. If s denotes the number of linear units in the base, then s can also denote the number of linear units in the height; hence the formula for the area of the square becomes:

$$A = s \times s.$$

This is commonly written

$$A = s^2 \text{ (read "the square of } s\text{")}.$$

EXAMPLE 1. Find the area of a square if each side is 8″.

SOLUTION. A (the number of square inches) = s^2 square inches.

Place 8 for s:

$A = 8^2$ square inches, or

$A = (8 \times 8)$ square inches, or

$A = 64$ square inches.

Ans. $A = 64$ sq. in.

EXAMPLE 2. Find the area of a square if each side is 5.4′.

SOLUTION. A (the number of square feet) = s^2 square feet.

(*Estimate:* 5×6, or 30, sq. ft.)

$A = 5.4^2$ square feet, or

$A = (5.4 \times 5.4)$ square feet, or

$A = 29.16$ square feet.

Ans. $A = 29.16$ sq. ft.

EXERCISES

1. Find the area of a square if each side is 12″.

2. Find the area of a square in square feet if each side is 1′ 6″.

3. Find the area of a square in square inches if each side is 1′ 2″. How many square feet in its area?

4. Copy and fill in the following table for areas of squares.

FORMULA: $A = s^2$

s	ESTIMATE	A
a. 7″		
b. 5.3″	5 × 6, or 30, sq. in.	
c $8\frac{1}{2}$″		
d. $6\frac{1}{4}$″		
e. 1′ 4″		sq. in.
f. 3′ 3″		sq. ft.
g. 12′ 6″		sq. ft.
h 2 yd. 9 in.		sq. yd.
i. 150 rd.		acres
j. 48.2′		sq. ft.

Express answers to f, g, h, i, j decimally.

5. The side of one square is $7\frac{1}{2}$″; the side of another square is $12\frac{1}{2}$″.

(a) Find their perimeters.

(b) Find the ratio of their perimeters.

(c) The first perimeter is what per cent of the second?

(d) Find their areas.

(e) Find the ratio of their areas.

(f) The first area is what per cent of the second?

(g) The second is how many square inches larger than the first?

(h) The second is what per cent larger than the first?

6. A square lot of land is 84′ long on each side. Draw a plan of the lot to the scale of 20′ to 1″. Find the area of the lot.

7. A square lot of land is 20 rods long on each side. How many square rods are there in its area? How many acres?

8. A man owns two lots of land, one of which is 40′ by 40′ and the other is 25′ by 64′.

(*a*) Find their areas.

(*b*) Find the ratio of their areas.

(*c*) Which would require the longer fence to inclose it? How much longer?

9. One lot of land is found to be 54′ by 24′. A second lot of land is found to be 36′ by 36′.

(*a*) Find their areas.

(*b*) Which would require the longer fence to inclose it? How much longer?

(*c*) The area of the square is what per cent of the area of the rectangle?

(*d*) The perimeter of the square is what per cent of the perimeter of the rectangle?

10. Find the side of a square whose area is 25 sq. in.

SOLUTION. Place 25 for A in the formula,

$$A = s^2.$$

Thus, $25 = s^2.$

Take the square root: $5 = s.$

Since s^2 means $s \times s$, you find one of the two equal factors of 25; that factor, 5, is called the ***square root*** of 25.

CHECK. $25 = 5^2$, or 5×5.

Ans. $s = 5''$.

11. Find the side of a square whose area is 144 sq. in.

SOLUTION. Place 144 for A in the formula,

$$A = s^2.$$

Thus, $144 = s^2.$

Take the square root: $12 = s$.

CHECK. $144 = 12^2$, or 12×12. *Ans.* $s = 12''$

12. Following the model in Exercises 10 and 11, find the required values in the following table. Copy the table and fill in the values.

FORMULA: $A = s^2$

	s	ESTIMATE	A
a.	4″	4″	16 sq. in.
b.			36 sq. ft.
c.			81 sq. in.
d.			49 sq. yd.
e.			64 sq. in.
f.			121 sq. in.
g.			400 sq. yd.
h.			1,600 sq. ft.
i.			196 sq. in.
j.			225 sq. ft.
k.			4,900 sq. ft.
l.			90,000 sq. ft.
m.			256 sq. rd.
n.			324 sq. ft.

13. The area of a square field is 900 sq. rd. How long is it? How long a fence will be required to inclose it?

14. A square picture contains 169 sq. in. What are the inside dimensions of the frame that contains the picture?

15. A trunk has a square end, and the area of the end is 576 sq. in. How long must the shortest strap that will go around the trunk be?

16. The area of a square is 225 sq. in.

(*a*) How long is its perimeter?

(*b*) A rectangle is 20″ long and has the same perimeter as the square. What is its width?

(*c*) Find the area of the rectangle.

(*d*) The area of the rectangle is how much smaller than the area of the square?

(*e*) The area of the rectangle is what per cent smaller?

59. Parallelograms. Figure 44 is a parallelogram having b for its base and h for its height (or altitude).

The height h divides the parallelogram into two parts marked I and II. Part I is a triangle. If triangle I is

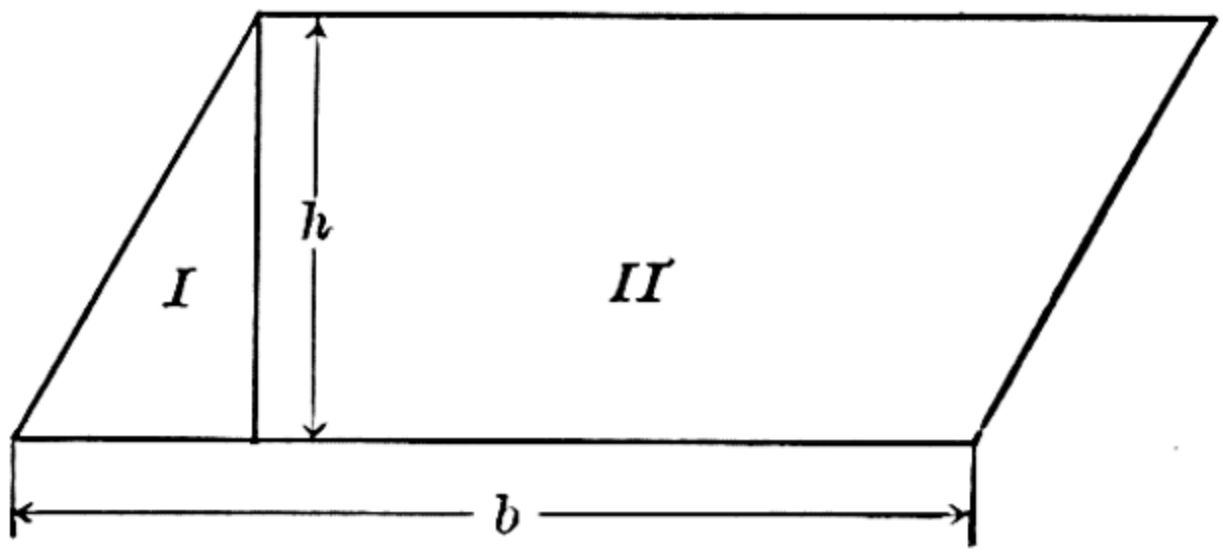


FIG. 44

taken from the position in Fig. 44 and placed in the position shown in Fig. 45, a rectangle is formed having the same base and altitude as the parallelogram in Fig. 44. The

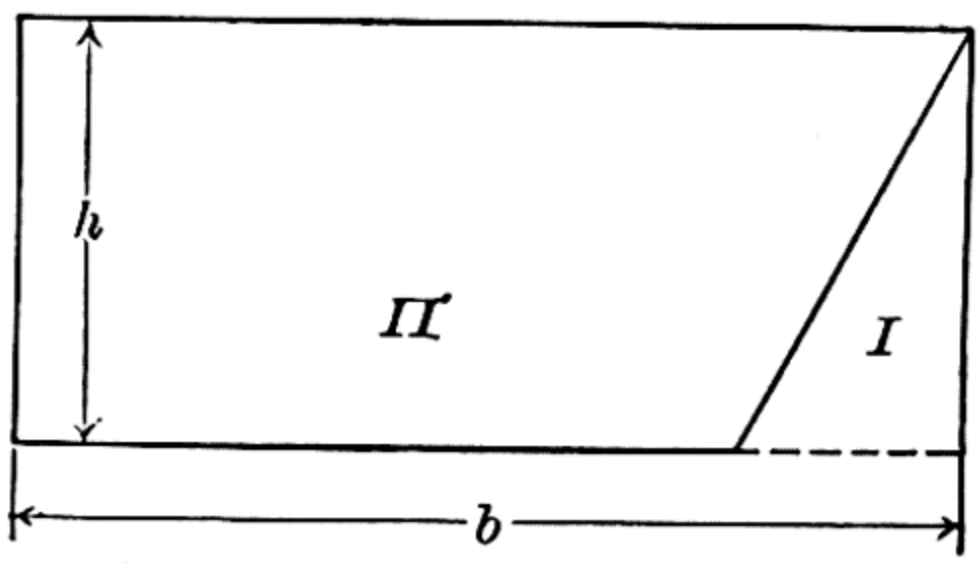


FIG. 45

two figures have the same areas, for they are made up of the same two parts I and II. For the rectangle we have:

A (the number of square units) = bh square units.

Hence, for the parallelogram we have:

A (the number of square units) = bh square units.

EXERCISES

1. Find the area of the parallelogram having its base $4\frac{1}{2}''$ long and its height $5''$.

2. Find the area of a parallelogram having its base $7\frac{1}{2}''$ long and its height $5\frac{1}{2}''$.

3. Two sides of a parallelogram are $2.3''$ and $1.8''$; they include an angle of $50°$. Draw the parallelogram. Draw the altitude.

(*a*) Measure the height to the nearest tenth of an inch.

(*b*) Find the area of the parallelogram.

4. Two sides of a parallelogram are $2\frac{1}{2}''$ and $2.1''$; they include an angle of $40°$. Draw the parallelogram. Draw the altitude.

(*a*) Measure the height to the nearest tenth of an inch.

(*b*) Find the perimeter of the parallelogram.

(*c*) Find its area.

5. Two sides of a parallelogram are $2''$ and $1.8''$; they include an angle of $35°$. Draw the parallelogram. Draw the altitude.

(*a*) Measure the height to the nearest tenth of an inch.

(*b*) Find the area of the parallelogram.

(*c*) Find the area of the rectangle having the same base and altitude as this parallelogram.

(*d*) If $2''$ and $1.8''$ were the sides of a rectangle, what would its area be?

6. Two sides of a parallelogram are $3.2''$ and $4.1''$; they include an angle of $120°$. Draw the parallelogram. Draw the altitude.

(*a*) Measure the height.

(*b*) Find the perimeter and the area of the parallelogram.

60. Triangles. Figure 46 is a triangle having b for its base and h for its height (or altitude).

Figure 47 is a parallelogram having the same base and altitude as the triangle in Fig. 46. The dotted line

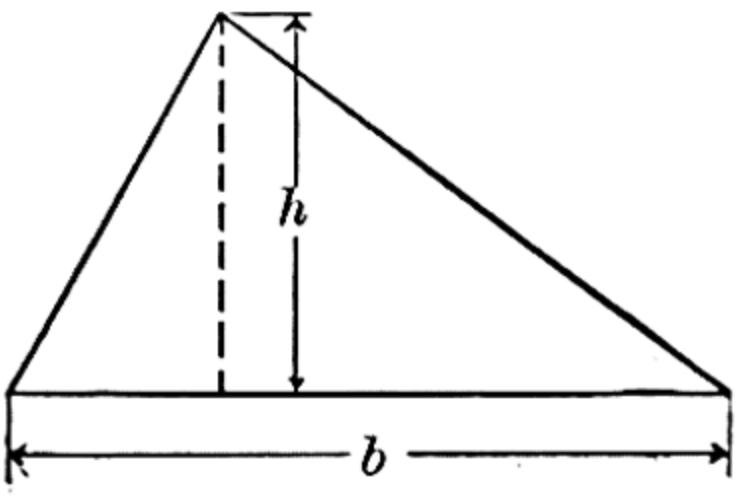


FIG. 46

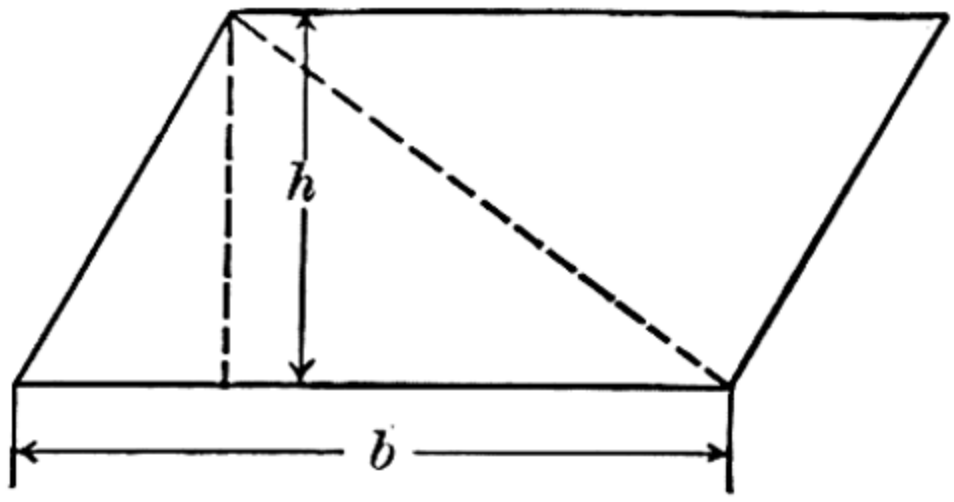


FIG. 47

divides the parallelogram into two triangles. If you make a tracing of the triangle in Fig. 46 and place it upon each of the triangles formed by the dotted line in Fig. 47, you will find that it exactly fits upon each one. Thus the triangle has an area equal to one half of the area of the parallelogram.

For the parallelogram we have:

A (the number of square units) $= bh$ square units.

Hence, for the triangle we have:

A (the number of square units) $= \frac{1}{2}\, bh$ square units.

In this formula $A = \frac{1}{2}\, bh$ (read $\frac{1}{2}$ the product of b and h), note that there are three distinct factors in the second member, $\frac{1}{2}$ and b and h.

EXAMPLE 1. Find the area of a triangle whose base is 8″ and height 6″.

SOLUTION. A (the number of square inches) $= \frac{1}{2}\, bh$ square inches.

Place 8 for b and 6 for h:

$A = \frac{1}{2}\,(8 \times 6)$ square inches, or

$A = 24$ square inches.

Ans. $A = 24$ sq. in.

NOTE. You will see that it makes no difference in the answer whether you multiply 8 by 6 and take one half of the product or multiply 8 by one half of 6.

EXAMPLE 2. The base of a triangle is $13\frac{1}{2}'$, and the height is $10.4'$; find its area.

SOLUTION. A (the number of square feet) $= \frac{1}{2}\,bh$ sq. ft.

(*Estimate:* 7×10, or 70, sq. ft.)

$A = \frac{1}{2}\ (13.5 \times 10.4)$ square feet, or

$A = 70.2$ square feet.

Ans. $A = 70.2$ sq. ft.

[Here it is easier to multiply 13.5 by one half of 10.4]

EXERCISES

Copy and fill in the following table for areas of triangles.

FORMULA: $A = \frac{1}{2}\ bh$

b	h	ESTIMATE	A
1. $8''$	$7''$		
2. $3'\ 6''$	$4'$		
3. $2'\ 3''$	$2'\ 6''$		
4. $5\frac{1}{2}''$	$6\frac{1}{2}''$	3×6, or 18, sq. in.	
5. $3'\ 6''$	$4'\ 6''$		sq. ft.
6. $3.7''$	$2.4''$		
7. $13.5'$	$8.6'$		
8. $7\frac{1}{4}''$	$12''$		
9. $1'\ 1''$	$9''$		sq. in.
10. $4\frac{3}{4}''$	$2\frac{1}{4}''$		

11. Find the area of a triangle if its base is $8\frac{3}{4}''$ and its height $4\frac{1}{2}''$.

12. Find the area of the gable end of a house in the form of a triangle whose base is $32'$ and height $12'$.

13. (*a*) Find the area of the lot of land represented by the triangle in Fig. 48.

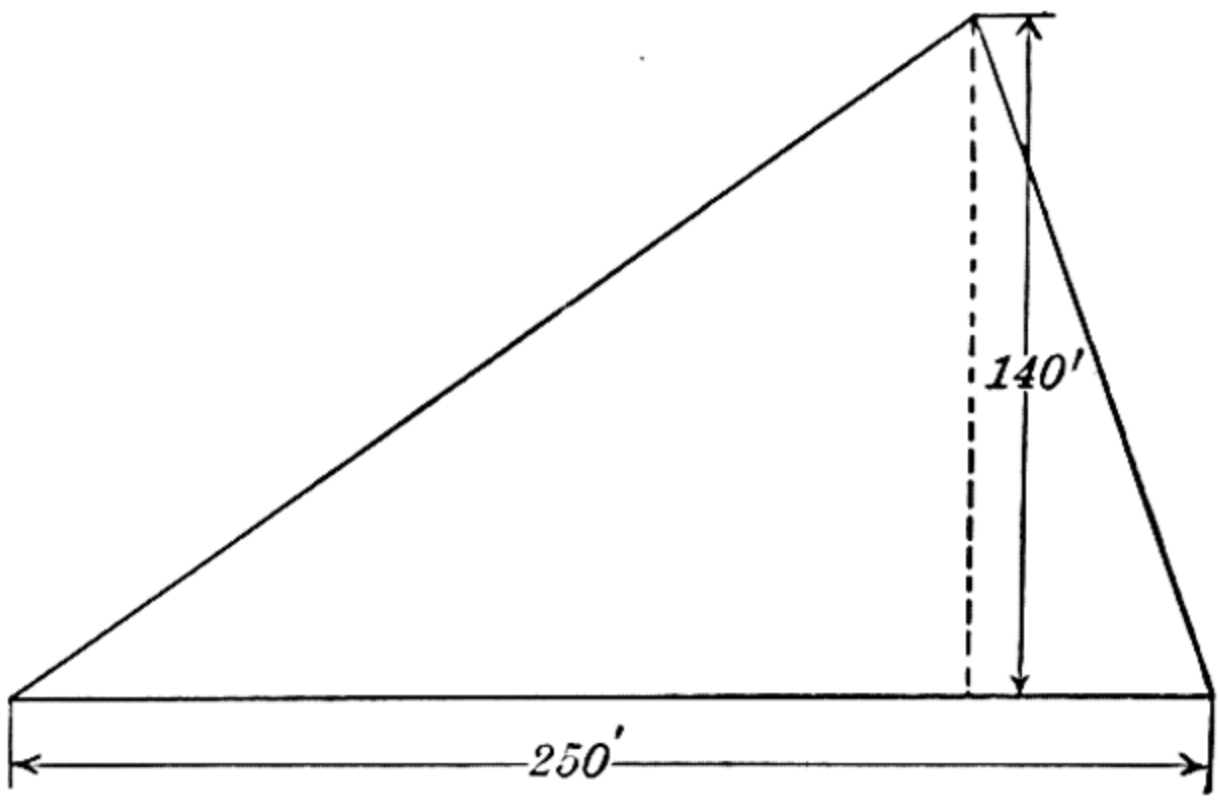


FIG. 48

(*b*) By measuring the base and height find the scale used in the drawing.

14. Figure 49 is the plan of a field drawn to a certain scale.

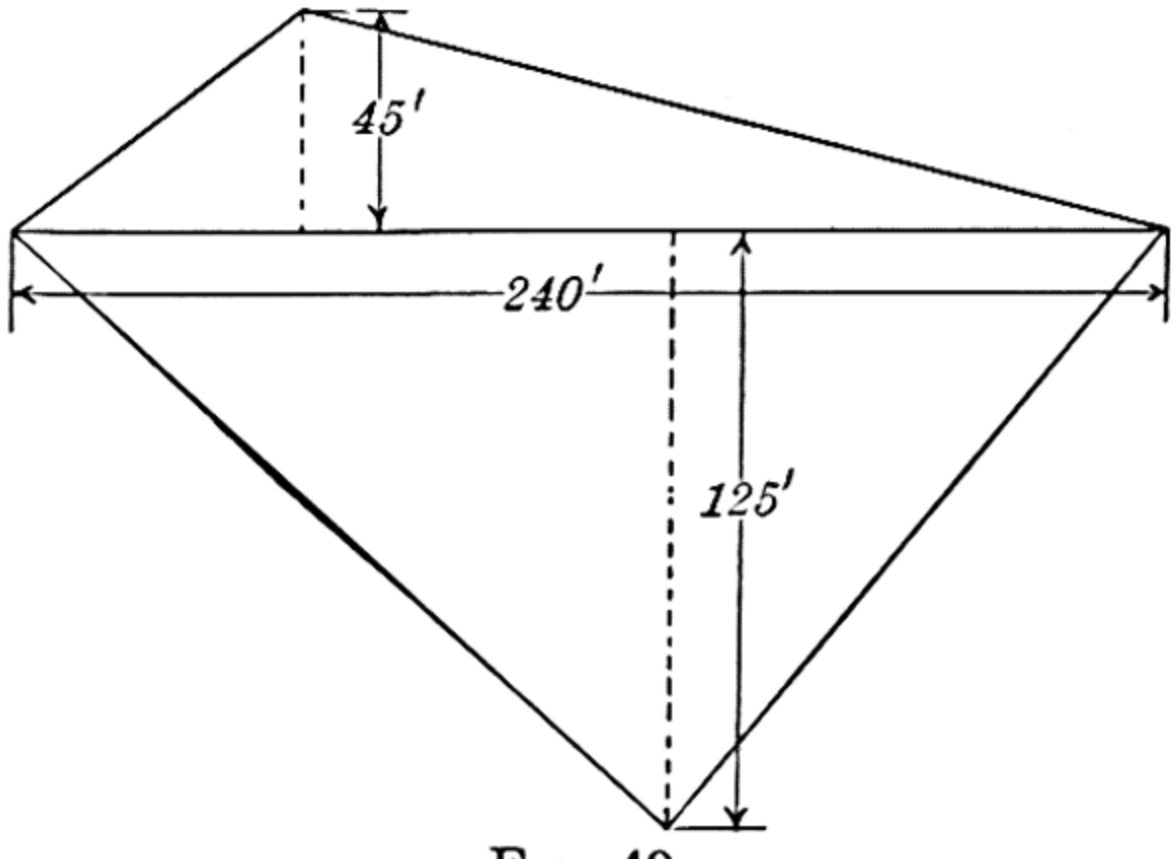


FIG. 49

(*a*) Find the scale used.
(*b*) Find the area of the larger triangle.
(*c*) Find the area of the smaller triangle.
(*d*) How many square feet are there in the field?

15. Figure 50 is the plan of a field. Find the area.

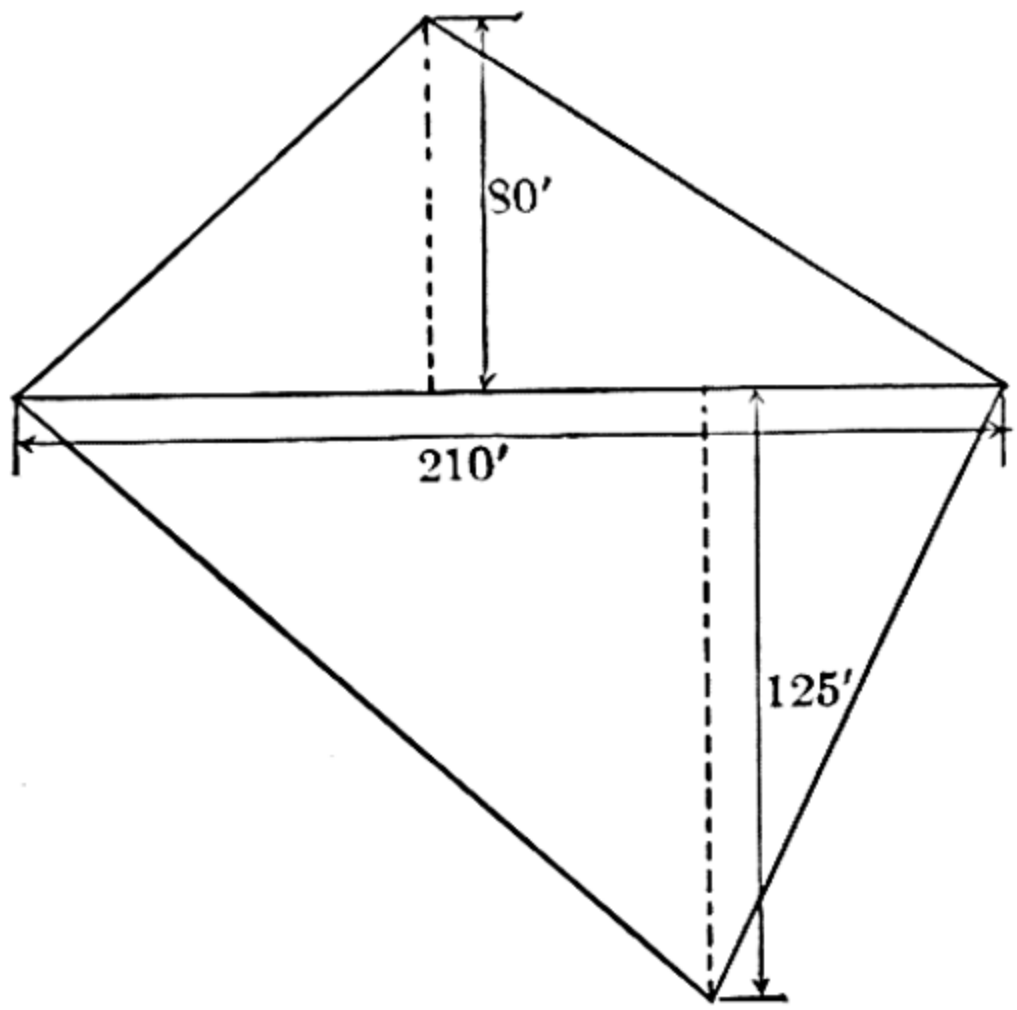


FIG. 50

16. The area of a triangle is 36 sq. ft. The base is 18′; find its height.

SOLUTION. Place 36 for A and 18 for b in the formula

$$A = \tfrac{1}{2}\, bh.$$

Thus, $36 = \tfrac{1}{2} \times 18\, h,$

or $36 = 9\, h.$

Divide each member by 9:

$4 = h.$ *Ans.* $h = 4'.$

CHECK. $36 = 9 \times 4.$

17. The area of a triangle is 120 sq. in. and its height is 18″. Find its base.

SOLUTION. Place 120 for A and 18 for h in the formula

$$A = \tfrac{1}{2}\, bh.$$

Thus, $120 = \tfrac{1}{2}\, b \times 18.$ (*Estimate* = 13″.)

Divide each member by 9:

$13\tfrac{1}{3} = b.$ *Ans.* $b = 13\tfrac{1}{3}''.$

18. Find the height of a triangle if its area is 40 sq. ft. and its base 20′.

19. Find the base of a triangle if its area is 35 sq. in. and its height 14″.

20. Find the base of a triangle if its area is $42\frac{1}{2}$ sq. ft. and its height 15′.

21. The area of a triangle is $15\frac{1}{2}$ sq. in. and its height is 20″. How long is its base?

22. The area of a triangle is $3\frac{3}{4}$ sq. in., and its height is 5″. How long is its base?

23. The base of a triangle is 10″. It has the same area as a square whose side is 6″. Find the height of the triangle.

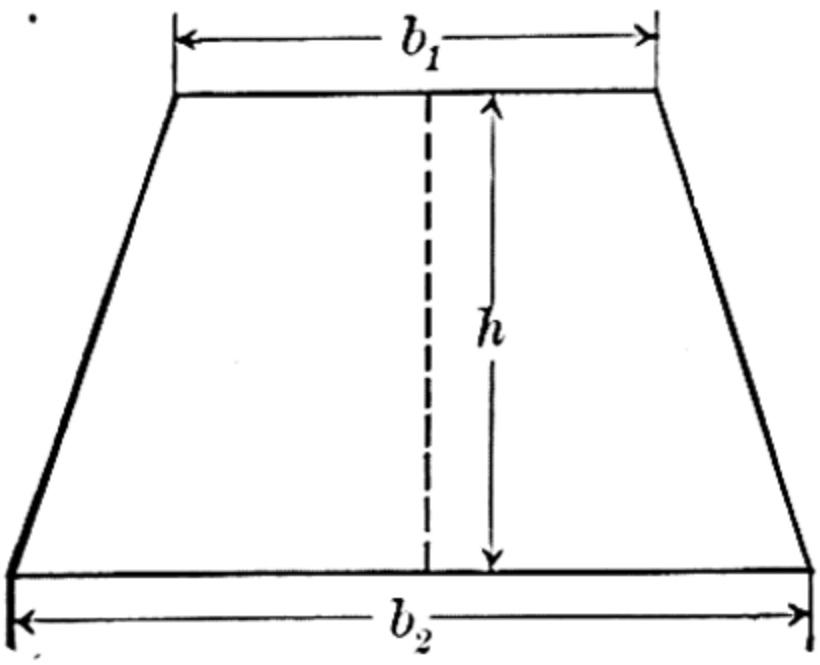


FIG. 51

61. Trapezoids. A *trapezoid* is a plane figure inclosed by four lines only two of which are parallel.

In the trapezoid of Fig. 51 the parallel sides, b_1 (read "b sub one") and b_2 (read "b sub two"), are the two *bases;* h, which is a line perpendicular to the bases, is the *height* (or *altitude*).

EXERCISES

1. Measure the sides and altitude of the trapezoid in Fig. 51 to the nearest *hundredth* of an inch.

(*a*) What is the sum of the two bases measured to the nearest hundredth of an inch?

(*b*) What is the perimeter?

(*c*) Measure the four angles with your protractor. What is the sum of the four angles?

(*d*) What is the ratio of the upper base to the lower base?

(*e*) b_2 is what per cent longer than b_1? (Answer to the nearest per cent.)

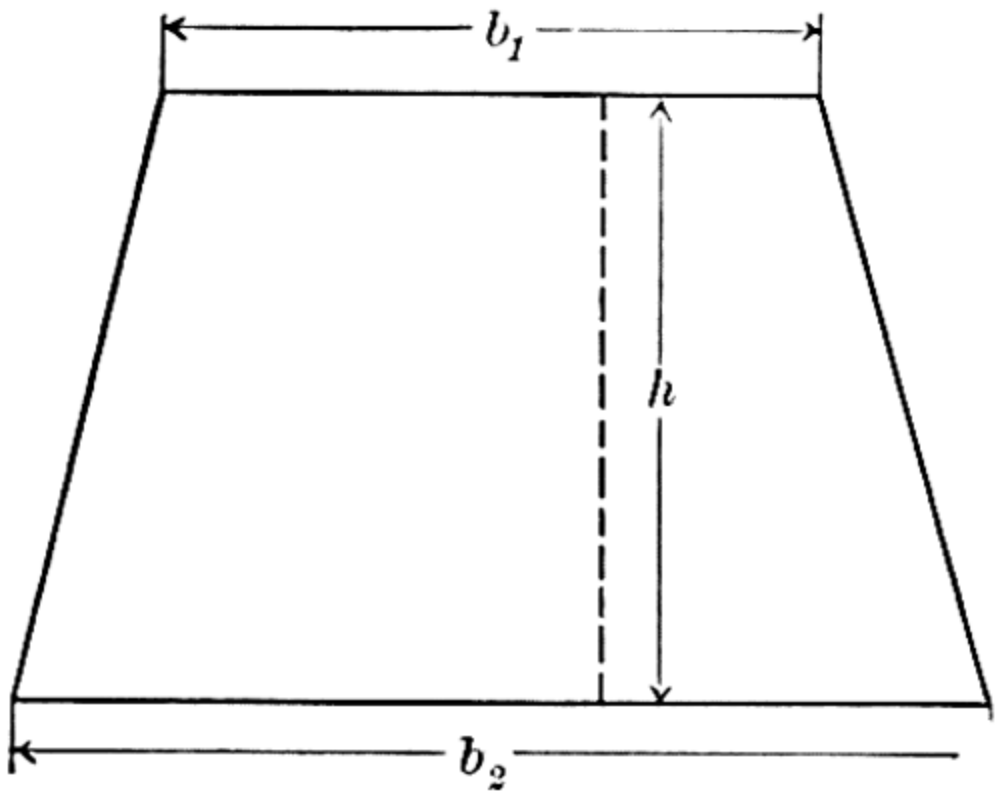


FIG. 52

2. Measure the sides and the altitude of the trapezoid in Fig. 52 to the nearest hundredth of an inch. Find all results required in Exercise 1.

62. Area of Trapezoid. Part *I* of Fig. 53 is a trapezoid having b_1 and b_2 for its bases and h for its altitude. If

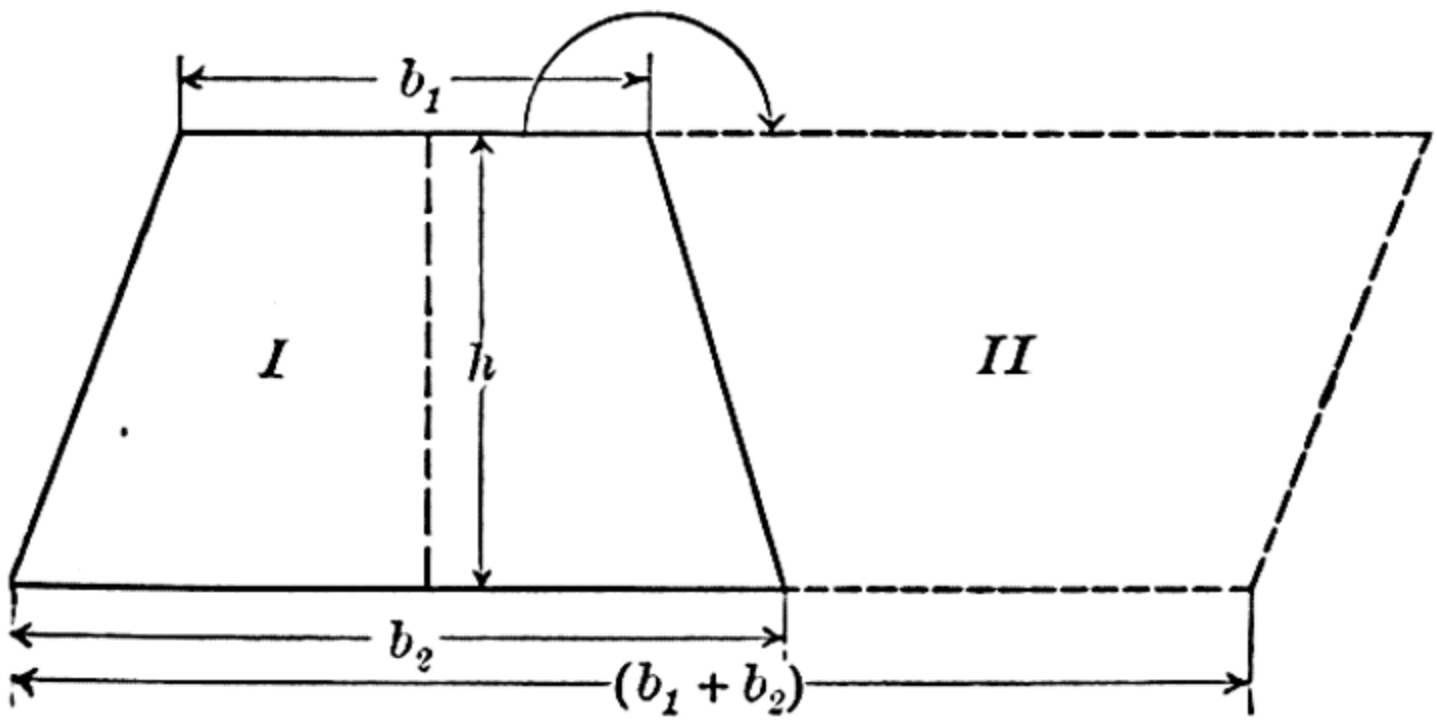


FIG. 53

trapezoid *I* is turned so that b_2 lies on b_1 extended, then a parallelogram is formed having $(b_1 + b_2)$ for its base and

h for its altitude. Note that trapezoid I takes the position of trapezoid II.

For the parallelogram we have:

A (the number of square units) $= (b_1 + b_2)\, h$ square units.

Hence, for the trapezoid, which is one half of the parallelogram, we have:

A (the number of square units) $= \frac{1}{2}(b_1 + b_2)\, h$ square units.

The formula $A = \frac{1}{2}(b_1 + b_2)\, h$ should be read "A equals one half the sum of b_1 and b_2 times h."

Note that there are three distinct factors: $\frac{1}{2}$ and $(b_1 + b_2)$ and h.

The signs of grouping ( ) are called *parentheses.*

EXAMPLE 1. Find the area of a trapezoid whose bases are $15''$ and $9''$ and whose height is $4\frac{1}{2}''$.

SOLUTION. A (the number of square inches) $= \frac{1}{2}(b_1 + b_2)\, h$ square inches.

Place 15 for b_1, 9 for b_2, and $4\frac{1}{2}$ for h:

$A = \frac{1}{2}(15 + 9) \times 4\frac{1}{2}$ sq. in., or (*Estimate* = 50 sq. in.)

$A = \frac{1}{2} \times 24 \times \frac{9}{2}$ sq. in., or

$A = 54$ sq. in.

Ans. $A = 54$ sq. in.

EXAMPLE 2. The bases of a trapezoid are $3\frac{1}{2}''$ and $4\frac{1}{2}''$. The height is $5\frac{1}{4}''$. Find its area.

SOLUTION. A (the number of square inches) $= \frac{1}{2}(b_1 + b_2)\, h$ square inches.

Place $3\frac{1}{2}$ for b_1, $4\frac{1}{2}$ for b_2, and $5\frac{1}{4}$ for h:

$A = \frac{1}{2}[(3\frac{1}{2} + 4\frac{1}{2}) \times 5\frac{1}{4}]$ sq. in., or (*Estimate* = 20 sq. in.)

$A = \frac{1}{2}(8 \times \frac{21}{4})$ sq. in., or

$A = 21$ sq. in. *Ans.* $A = 21$ sq. in.

EXERCISES

Copy and fill in the following table for areas of trapezoids.

FORMULA: $A = \frac{1}{2}(b_1 + b_2)h$

b_1	b	h	ESTIMATE	A
1. 8″	6″	5″	35 sq. in.	35 sq. in.
2. 13″	9″	$4\frac{1}{2}$″	50 sq. in.	
3. $15\frac{1}{2}$″	$12\frac{1}{2}$″	$5\frac{1}{2}$″		
4. 5.4″	3.5″	2.8″		
5. 142′	133′	92′		
6. 16.4′	32.5′	20.7′		
7. 1′ 6″	1′ 3″	2′		
8. 8.7″	4.9″	6.7″		

9. Find the area of a lot of land the shape of a trapezoid if the parallel sides are 240′ and 80′ and the altitude is 125′. How many acres are there in the lot?

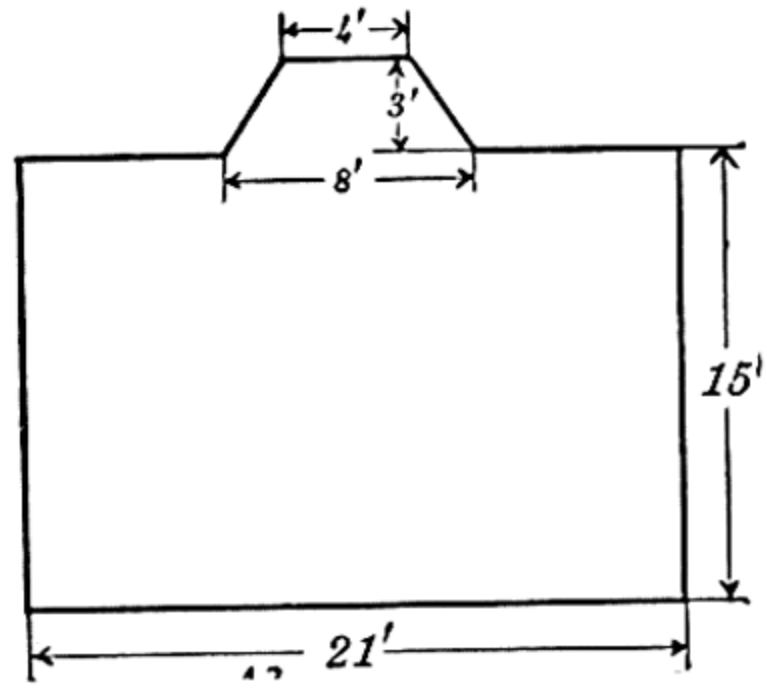


FIG. 54

10. Find the area of the floor of which this diagram (Fig. 54) is a plan. (The scale is 15′ to the inch.)

II. Representation of Number Data by Rectangular Graphs

63. The following examples illustrate the use of rectangular graphs to represent quantities.

Example 1. The average annual production of wheat in the six leading countries in the 20-year period from 1895 to 1914 is represented by the rectangular graphs in Fig. 55.

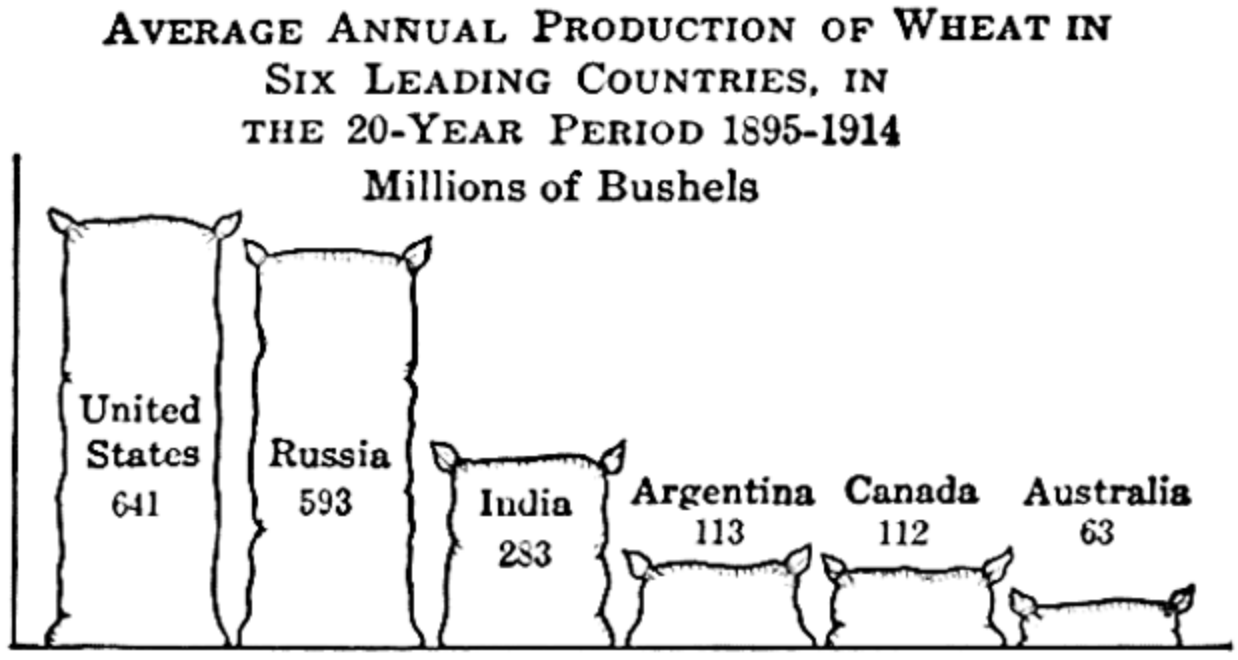


Fig. 55

After inspecting this illustration, would you agree with the following statements? "The United States and Russia were running a close race before the war in the production of wheat. India was third with a production of less than one half that of the United States while Australia was lowest with a production of less than one tenth that of the United States."

Write in figures the number of bushels produced by each country. For example, Australia, 63,000,000 bushels.

How many bushels did these six countries produce?

These numbers are *round numbers*, or *approximate numbers*. See page 128.

EXAMPLE 2. The value of farm land per acre from 1850 to 1920 in the states of New York and Iowa, is shown by the rectangular graphs in Fig. 56.

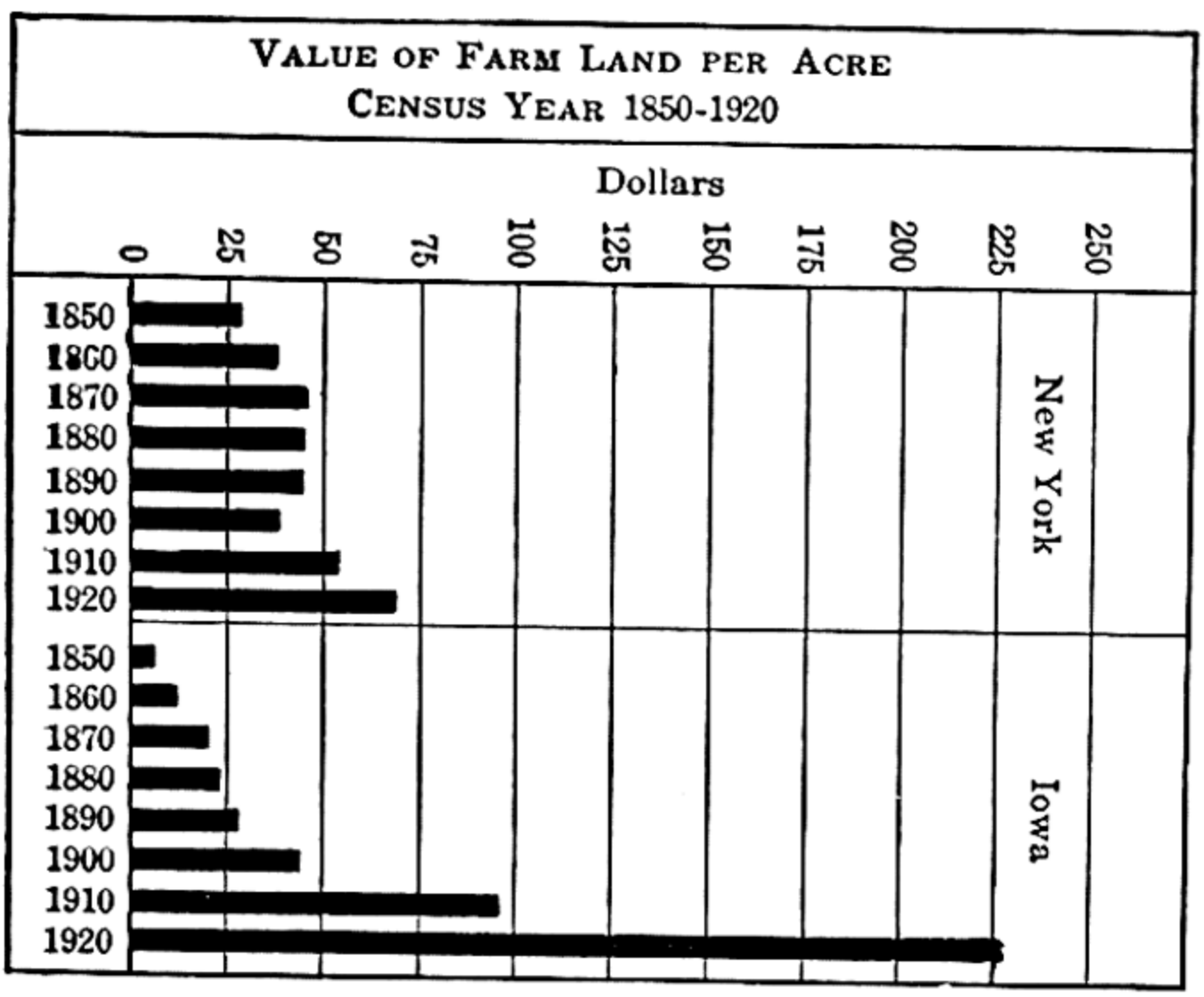


FIG. 56

After inspecting these graphs, would you agree with the following statements? "In 1900 there was not much difference in the value of an acre of land in Iowa and in New York. In 1910 an acre of land in Iowa was worth nearly twice as much as an acre of land in New York, and in 1920 an acre of land in Iowa was worth more than three times as much as an acre in New York."

How did the values of land compare in the two states in 1850? In 1860? In 1870? In 1880?

The value of an acre of land in New York State in 1920 was how many times its value in 1850?

The value of an acre of land in Iowa in 1920 was how many times its value in 1850?

EXAMPLE 3. The United States Bureau of Education reported in 1920 that out of every 1000 students entering high schools in the United States 725 stay into the second year, 525 into the third year, 449 into the fourth year, and 418 are graduated. These facts are shown by the rec-

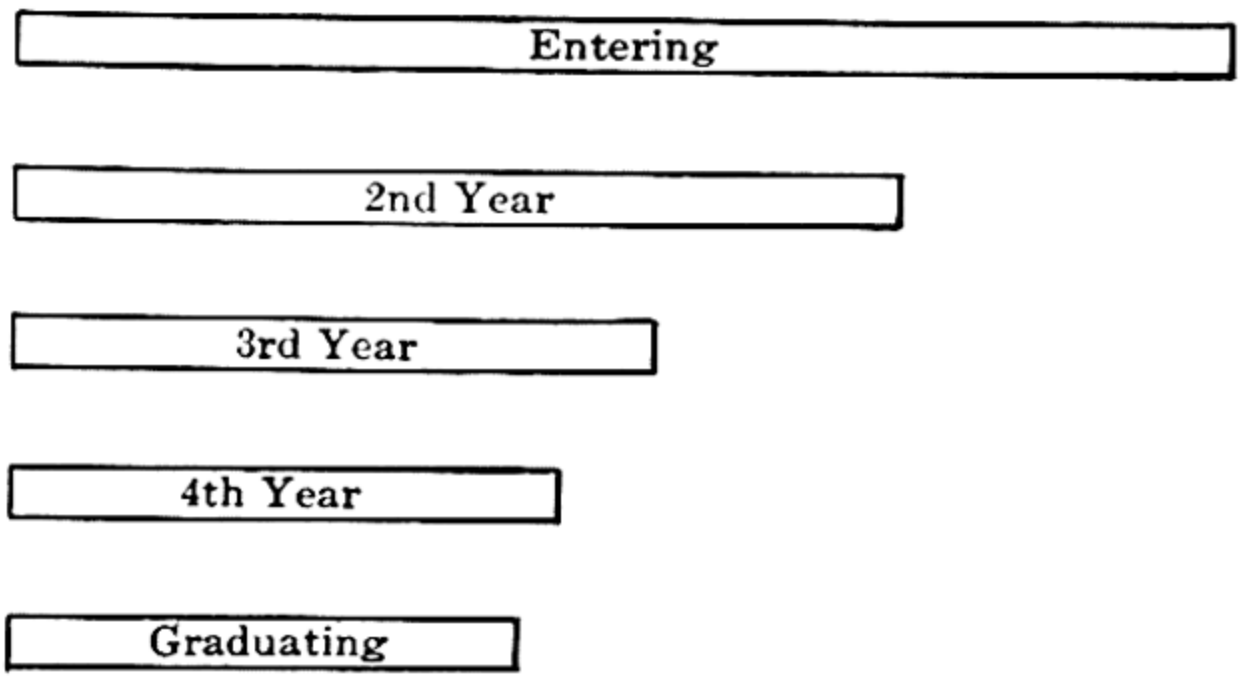
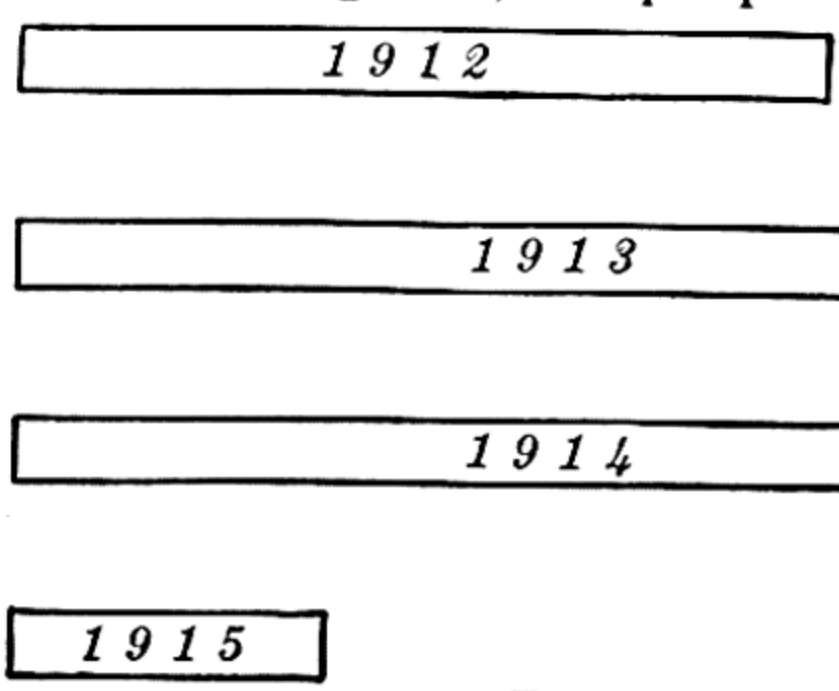


FIG. 57

tangular graphs in Fig. 57. Measure the lengths of the rectangles, determine the scale that was used, and state whether the facts are correctly represented or not.

EXAMPLE 4. The number of immigrants to the United States in 1912, 1913, 1914, and 1915 respectively is repsented by the following rectangular graphs, 1″ of length representing 500,000 people.



1912: 1.68″
1.68 × 500,000
1913: 2.40″
2.40 × 500,000
1914: 2.44″
2.44 × 500,000
1915: 0.65″
0.65 × 500,000

FIG. 58

Find the number of immigrants for each of the years, 1912, 1913, 1914, and 1915.

EXERCISES

For these exercises it will be necessary to use cross-section paper.

1. The energy needs for different individuals in terms of hundred-calorie portions are as follows:

A man or boy using much muscular energy in work or play needs 40
A woman or girl using much muscular energy in work or play needs 27
A boy or girl between 10 and 12 years of age needs 20
A boy or girl between 6 and 9 years of age needs 17
A boy or girl between 2 and 5 years of age needs 14

Represent these facts by rectangular graphs, letting one unit represent the hundred-calorie portion.

2. The fuel value of food is measured in calories, just as length is measured in inches. For convenience in planning meals, the hundred-calorie portion is sometimes used as a basis.

The fuel value of a few commonly used foods is given below where the unit is the hundred-calorie portion:

Beef, average, 10 per pound. (That is, 10×100 calories.)
Bread, 12 per pound. (That is, 12×100 calories.)
Eggs, about 5 per pound. (That is, 5×100 calories.)
Lettuce, $\frac{3}{4}$ per pound. (That is, $\frac{3}{4} \times 100$ calories.)
Milk, whole, 3 per pound. (That is, 3×100 calories.)
Sugar, granulated, 18 per pound. (That is, 18×100 calories.)

Represent these facts by rectangular graphs, selecting what seems to you to be a suitable unit for the hundred-calorie portion.

3. The number of immigrants to the United States in the years 1916 to 1923 inclusive is given below. After studying Exercise 4, page 156, represent these facts by rectangular graphs.

Year	Number	Year	Number
1916	299,000	1920	430,000
1917	295,000	1921	805,000
1918	111,000	1922	310,000
1919	141,000	1923	523,000

These numbers are round numbers, or approximate numbers.

4. By rectangular graphs compare the populations (1920) of the following cities in the United States:

New York	5,620,048	Cleveland	796,841
Chicago	2,701,705	St. Louis	772,897
Philadelphia	1,823,779	Boston	748,060
Detroit	993,678	Baltimore	733,826

(Let 1″ of length represent 1,000,000. First divide each number by 1,000,000; then draw the graphs.)

New York: $\frac{5,620,048}{1,000,000} = 5.62$ Cleveland: 0.80

St. Louis: 0.77

Detroit: $\frac{993,678}{1,000,000} = 0.99$ Boston: 0.75

5. The number of pupils enrolled in the schools of the United States in 1920 was 21,732,340, and the average daily attendance was 16,248,997. Represent these numbers by rectangular graphs, letting 1″ of length represent 10,000,000. (First write each number to the nearest 10,000,000.)

6. Figure 59 represents a graph from the 1922 *Yearbook* of the United States Department of Agriculture. The Government uses graphs such as this throughout its Re-

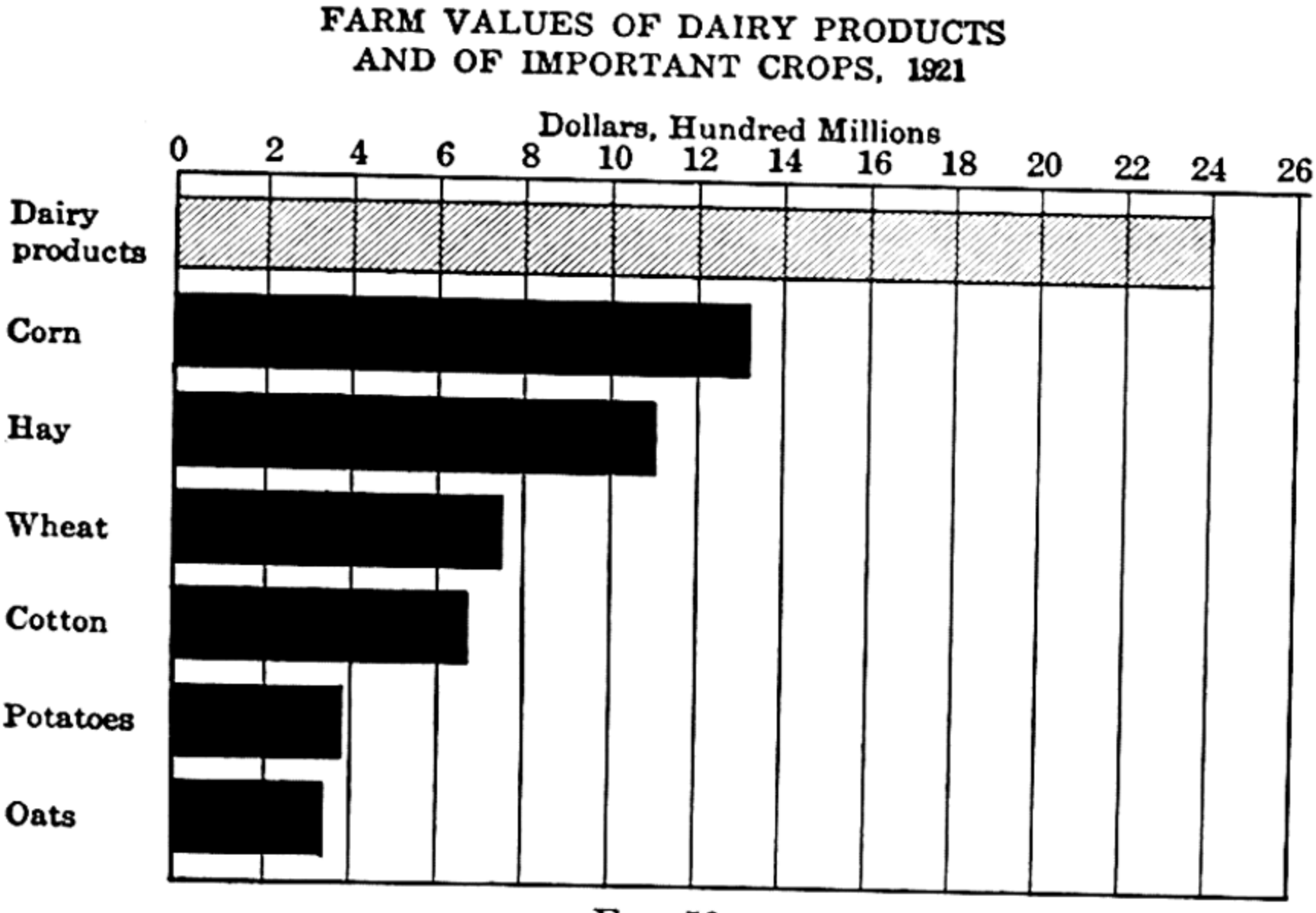


FIG. 59

ports in order to present the facts more clearly and convincingly than it could by tables of such facts. Tables of facts are usually dry and hard to read.

By careful measurement determine as accurately as you can the scale to which this graph is drawn. For example, the value of the corn crop was $1,297,213,000. The value of the wheat crop was $754,834,000. The value of the oat crop was $325,954,000. Determine the values of the other products listed. What per cent of the value of all the products listed was the value of the dairy products? Write out the complete story of this graph and compare the effect of "words" with the effect of a "picture."

III. Circles

64. A ***curved line*** is a line no part of which is straight.

A ***circle*** is a curved line all points of which are equally distant from a point within called the ***center.*** Any part of the curve is an ***arc.*** The ***radius*** of a circle is a straight line drawn from the center to any point in the curve. The *diameter* is a straight line joining two points of the curve and passing through the center.

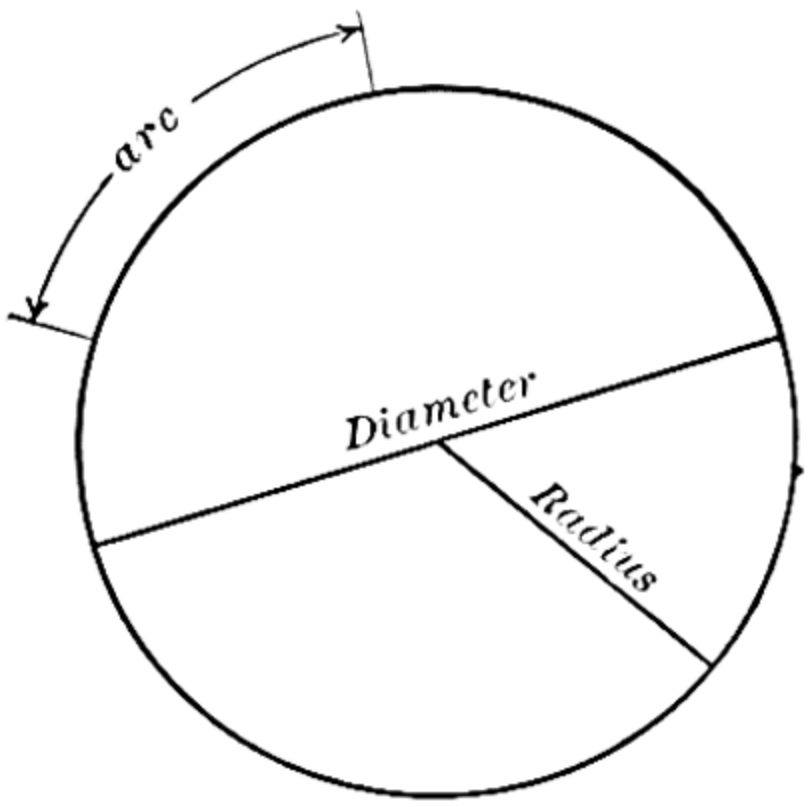


Fig. 60

Sometimes the surface inclosed by the curved line is called the circle, and the line itself is called the *circumference.*

The ***circumference*** of the circle is the length of the circle.

How do the radii (plural of radius) of a circle compare in length? What is the ratio of the diameter of a circle to its radius?

Using a tape, find the distance around a tomato can (or any other object of the same shape). Measure the diameter of the bottom of the can. Divide the number of inches in the circumference by the number of inches in the diameter. What is the ratio?

You find that the circumference of the bottom is a little more than 3 times as long as the diameter. If you were able to find how many times the circumference contained the diameter more accurately, you would find it to be 3.1416, or nearly $3\frac{1}{7}$ times. This mixed decimal 3.1416

is commonly expressed by the symbol π (read *pi*). Hence, to find the circumference of a circle multiply the diameter by π.

EXERCISES

1. Write the formula for the circumference of a circle when the diameter is given. Explain each letter in the formula.

2. Write the formula for the circumference of a circle when the radius is given.

65. Area of a Circle. The area of the circle is the number of square units of surface inclosed by the circle.

Cut the surface inclosed by a circle into any number of equal parts, say 16 as shown in Fig. 61. Fit them together as shown in Fig. 62. This figure resembles a parallelogram having for its base one half the circumference of a circle ($\frac{1}{2}$ *c*), and for its height the radius of the circle.

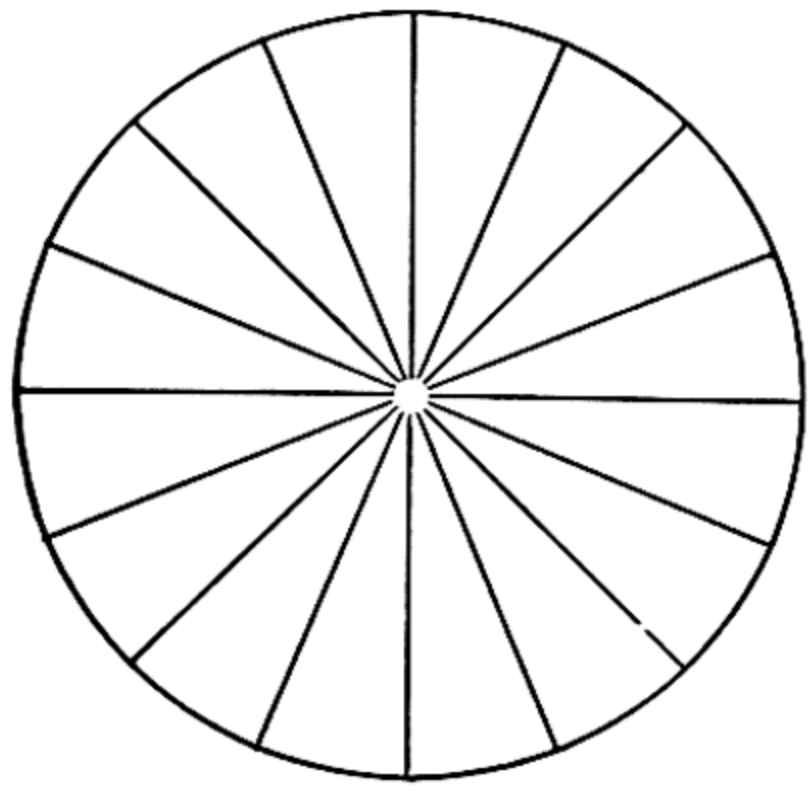


FIG. 61

For the *parallelogram* we have:

A (the number of square units) = ***bh*** square units.

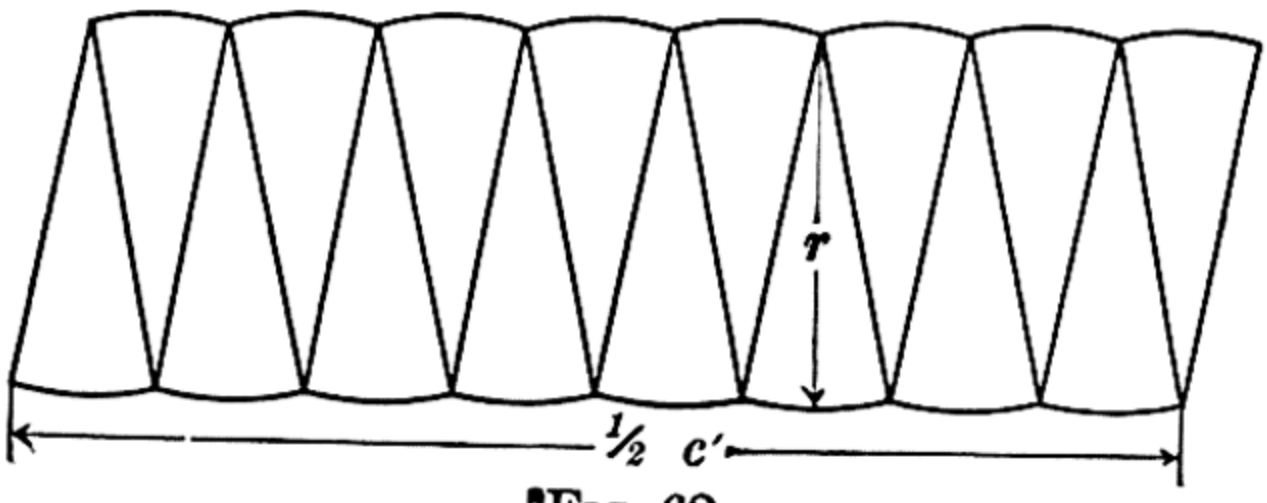


FIG. 62

Hence, for the circle we have:

A (the number of square units) $= \frac{1}{2}\,cr$ square units, where c = the circumference of the circle (in linear units), and r = the radius (in linear units).

But $c = 2\pi r$,

whence $A = \frac{1}{2} \times 2\pi r \times r$,

or $A = \pi r^2$,

where $\pi = 3.14159 \cdots$

In Fig. 63 there are four squares formed having r for each side; hence their combined area will be $4r^2$. It will be

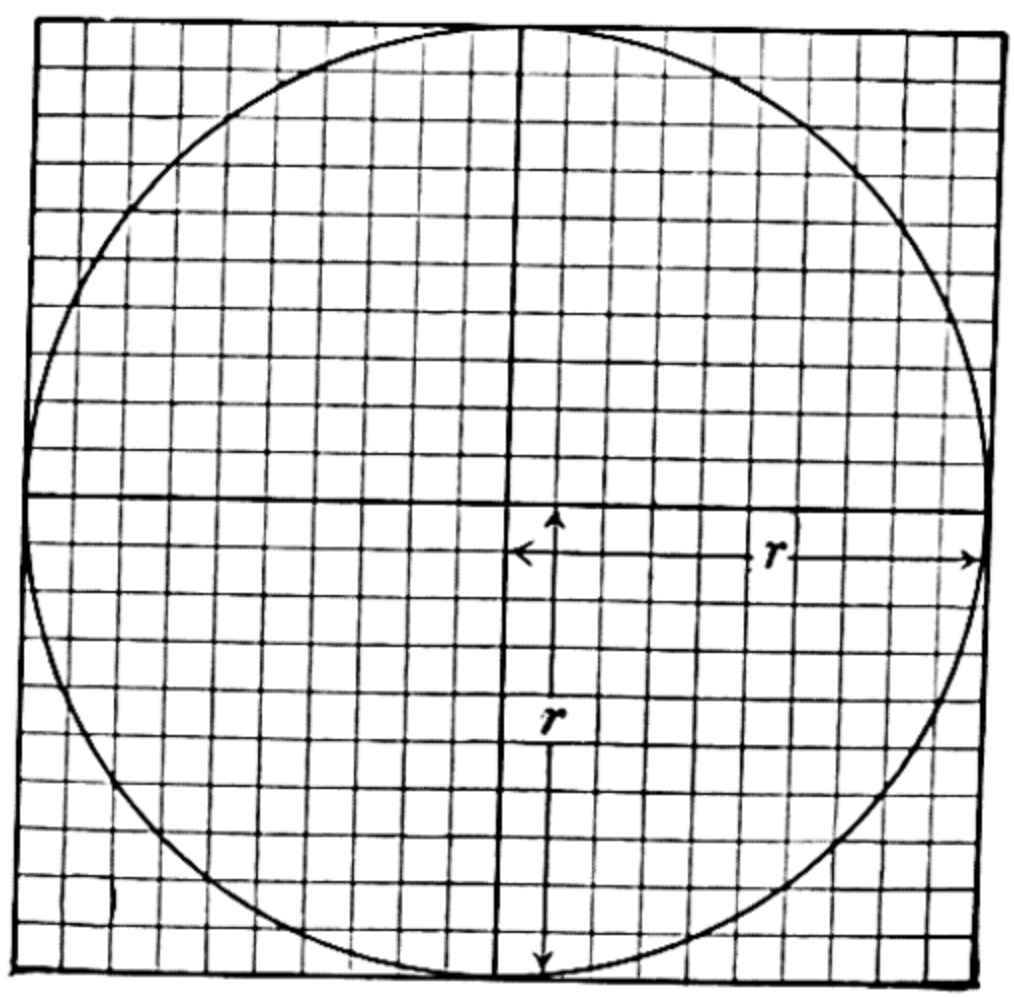


FIG. 63

seen that the area of the circle is somewhat less than $4r^2$. The circle drawn has a radius of one inch. Counting the small squares (hundredths of a square inch) in one quarter of the circle,we find that there are between 75 and 80. Multiplying this by 4, we get between 3 and 3.2 sq. in. for the area of the circle.

EXAMPLE 1. The radius of a circle is $2\frac{1}{4}''$. Find the circumference and area of the circle to three figures.

(The value $\pi = 3.14$ will be sufficiently accurate for our purpose here.)

SOLUTION.

$c = 2\pi r$, or

$c = 2 \times 3.14 \times 2.25''$, or (*Estimate* = 14″.)

$c = 14.13''$

Ans. $c = 14.1''$.

$A = \pi r^2$, or

$A = 3.14 \times (2\frac{1}{4})^2$ sq. in., or (*Estimate* = 16 sq. in.)

$A = 15.89$ sq. in.

Ans. $A = 15.9$ sq. in.

EXERCISES

Copy the following table and fill in the required values to three figures.

d	r	EST. c	$c = 2\pi r$	EST. A	$A = \pi r^2$
1. 6.2″	3.1″	19″	19.5″	30 sq. in.	30.2 sq. in.
2.	6.3″				
3.	4.4″				
4. 12.8′					
5.	4.9″				
6.	31.3′				
7. $7\frac{3}{4}''$					
8.	$1\frac{3}{8}''$				
9. $12\frac{1}{4}''$					

10. The diameters of iron pipes are as follows. Find their circumferences and the areas of their cross sections, using the formulas: $c = \pi d$ and $A = \frac{\pi d^2}{4}$.

(*a*) $d = 1''$	(*f*) $d = 1.5''$	(*k*) $d = 3.5''$	(*p*) $d = 2'$
(*b*) $d = 1/2''$	(*g*) $d = 1.75''$	(*l*) $d = 4''$	(*q*) $d = 2.5'$
(*c*) $d = 3/4''$	(*h*) $d = 2''$	(*m*) $d = 5''$	(*r*) $d = 3'$
(*d*) $d = 7/8''$	(*i*) $d = 2.5''$	(*n*) $d = 6''$	(*s*) $d = 3.5'$
(*e*) $d = 1.25''$	(*j*) $d = 3''$	(*o*) $d = 12''$	(*t*) $d = 6'$

11. The circumference of a circle is $11\frac{1}{2}''$ long. Find its radius to three figures.

SOLUTION. Place 11.5 for c and 3.14 for π in the formula

$$c = 2\pi r.$$

Thus, $11.5 = 2 \times 3.14\, r.$ (*Estimate* $= 1\frac{3}{4}''$.)

Divide each member by 6.28:

$$1.831 = r. \qquad Ans.\ r = 1.83''.$$

12. The circumference of a circle is 47.4′. Find its diameter to three figures.

13. The circumference of a circle is 134′. Find its radius to three figures.

14. The diameter of a circular grass plot is 42 feet.

(*a*) Find the number of feet around it.

(*b*) Find the number of square feet of turf in it.

(*c*) A 3-foot walk surrounds it. Find the number of square feet covered by the grass and walk.

(*d*) Find the area of the walk.

(*e*) The area of the walk is what per cent of the total area?

15. A circular table top has a diameter of 30″. A square table top has sides 30″ long.

(*a*) Find the perimeter of the top of each table.

(*b*) Find what ratio the perimeter of the circular top nas to the perimeter of the square top.

(*c*) Find what per cent the area of the circular top is of the area of the square top.

IV. Representation of Number Data by Circular Graphs

66. The unit of measure for a circle is an arc of one *degree*. Every circle contains 360 of these one-degree arcs.

To represent number data by circular graphs you must find out what part of a circle or how many degrees each number represents.

A circular graph can only be made when you have the total of the groups to be represented.

Example 1. The tax rate for a certain city for 1916 was \$17.80 per \$1000 valuation. This circular graph represents how it was divided. Find the approximate amount per \$1000 valuation for each purpose named.

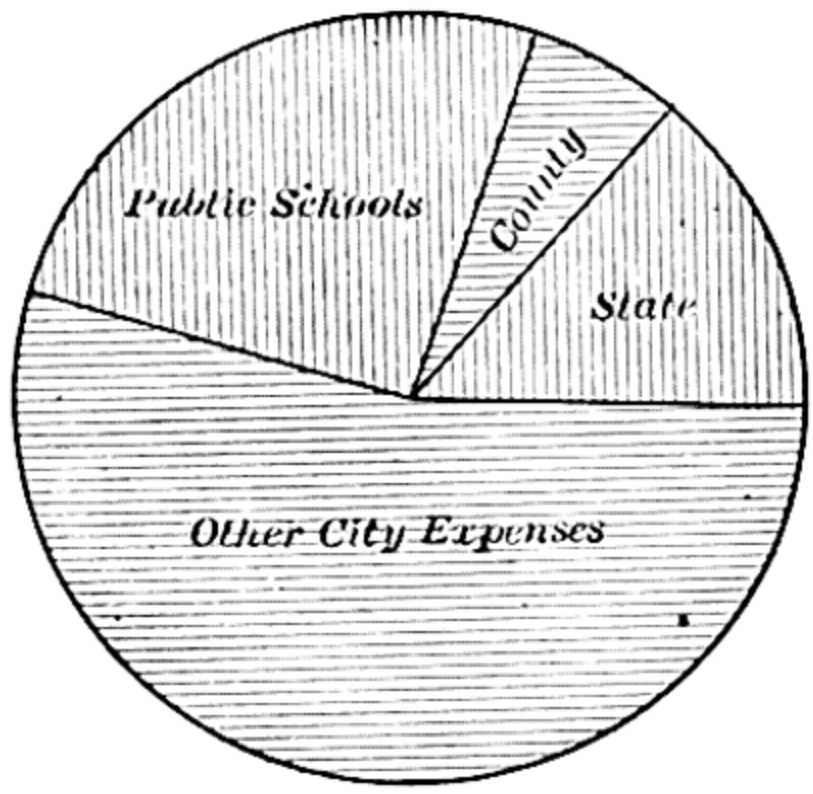


Fig. 64

Solution.

With the protractor find the number of degrees in each angle. The number of degrees in the arc will be the same.

State tax:	50°; $\frac{50}{360}$ of \$17.80 =	\$2.47.
County tax:	24°; $\frac{24}{360}$ of \$17.80 =	\$1.19.
Public schools:	90°; $\frac{90}{360}$ of \$17.80 =	\$4.45.
Other city expenses:	196°; $\frac{196}{360}$ of \$17.80 =	\$9.69.
Circle:	360°; total tax =	\$17.80.

EXAMPLE 2. Represent by a circular graph the following data from the United States Census, 1910:

Native-born whites of native parentage	49,488,575
Native-born whites of foreign parentage	18,897,837
Foreign-born whites	13,345,545
Negroes	9,827,763
All others	412,547
Total population	91,972,267

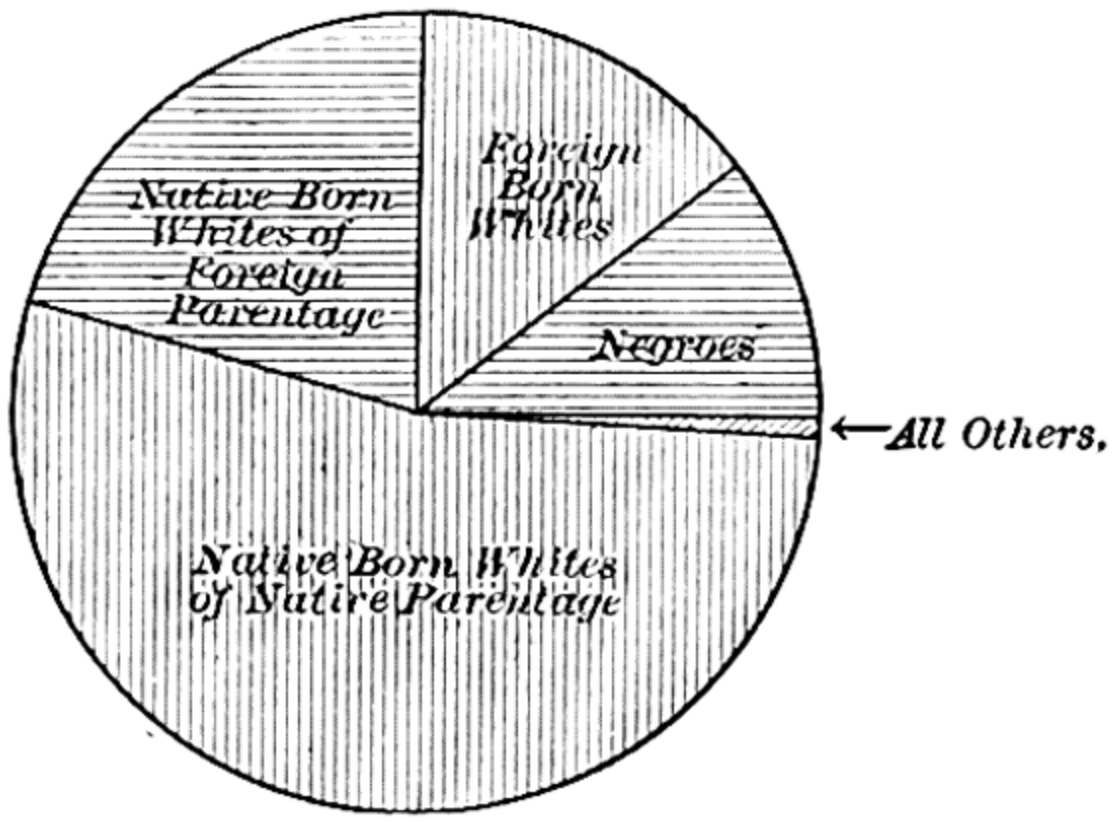


FIG. 65

$\frac{49,488,575}{91,972,267} = .538$ (nearest thousandth) .538 of 360° = 194°.

$\frac{18,897,837}{91,972,267} = .205$ (nearest thousandth) .205 of 360° = 74°.

$\frac{13,345,545}{91,972,267} = .145$ (nearest thousandth) .145 of 360° = 52°.

$\frac{9,827,763}{91,972,267} = .107$ (nearest thousandth) .107 of 360° = 38°.

$\frac{412,547}{91,972,267} = .005$ (nearest thousandth) .005 of 360° = 2°.

CHECK. 1.000 360°.

EXERCISES

1. The circular graphs in Fig. 66 are used by the United States Department of Agriculture in its *Yearbook* for 1921 to tell the story of the wasteful cutting of our virgin forests. Using your protractor to find the number of degrees in each angle, state whether the divisions are correctly represented or not.

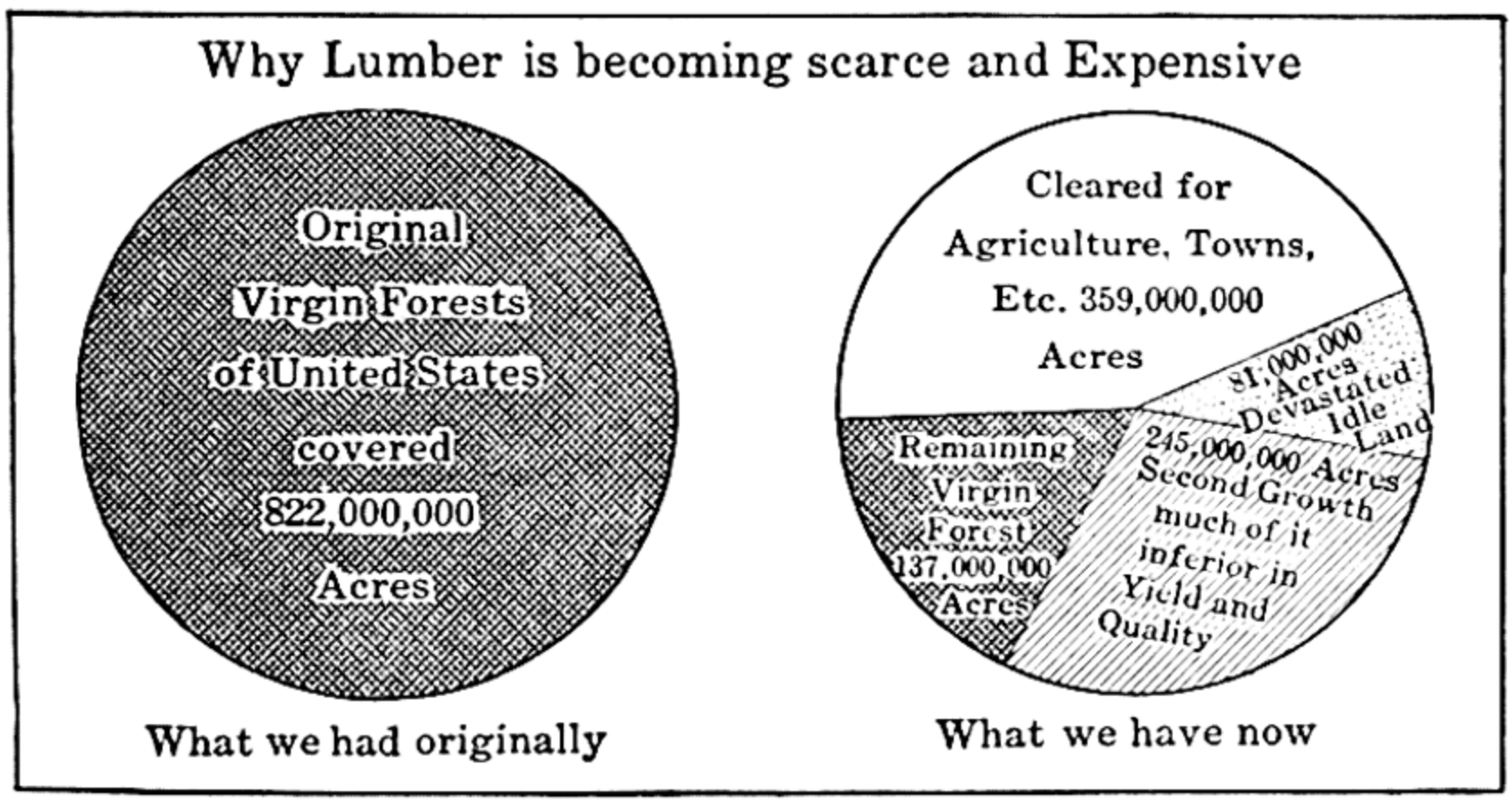


FIG. 66

2. The United States Department of Commerce analyzed American expenditures abroad by the circular graph in Fig. 67. Using your protractor to measure the angles, determine whether the divisions are correctly represented or not.

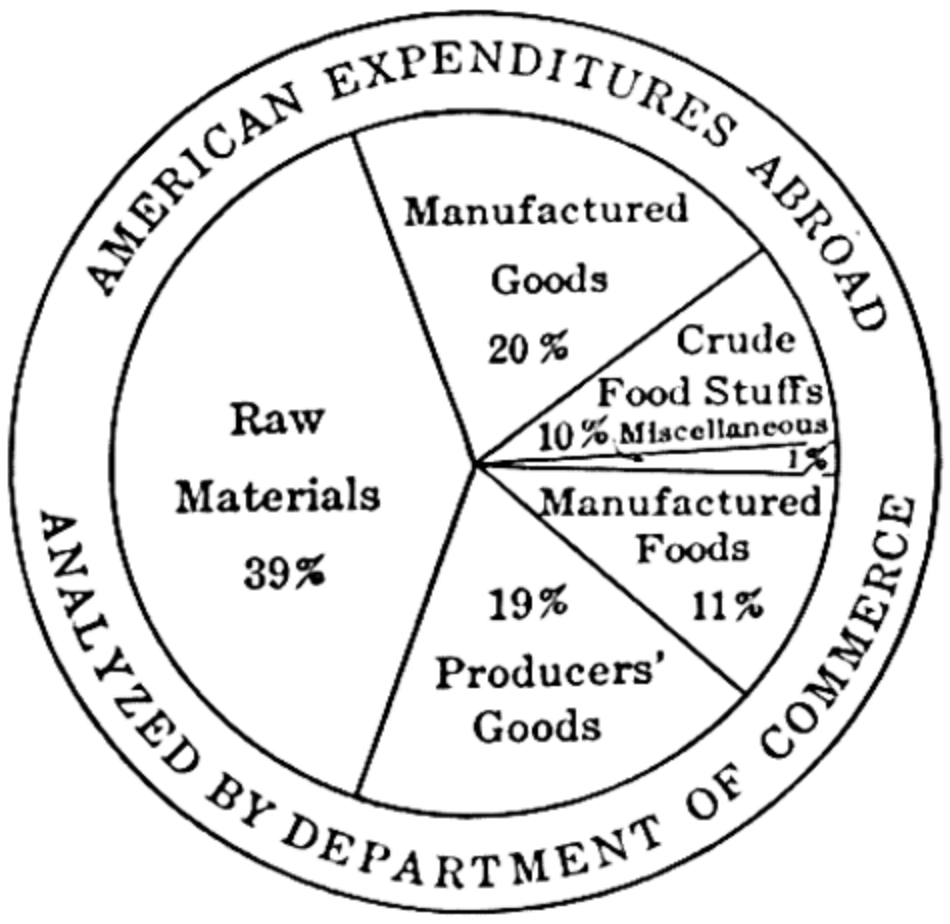


FIG. 67

3. From the graphs in Fig. 68 state what America's share in Japanese commerce is.

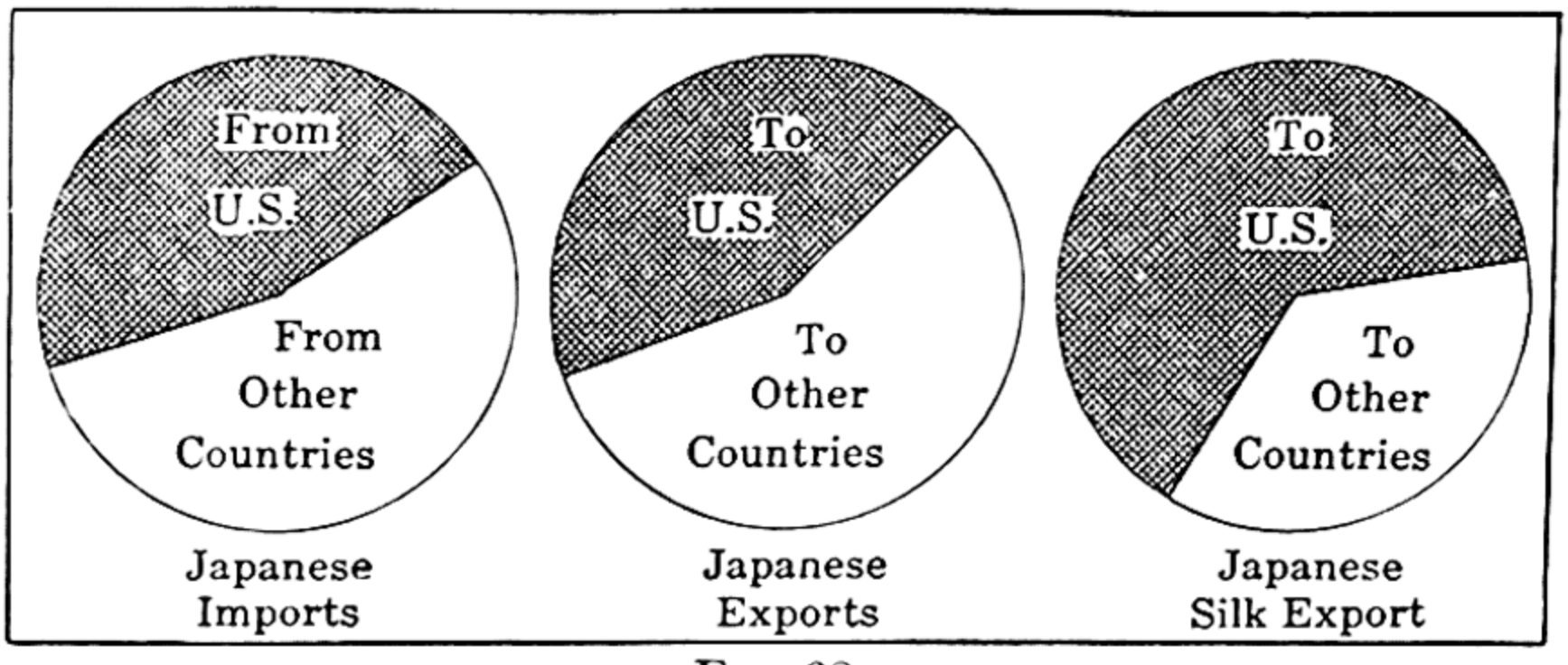


FIG. 68

4. In connection with the graph shown in Fig. 69 the following statement is made:

"Forty per cent of our people live outside incorporated places, practically all in the open country. Over 8 per cent more live in villages of less than 2,500 population, mostly retired farmers or tradesmen who are dependent upon the farmers for support. Nearly half of our population is agricultural or directly dependent upon agriculture."

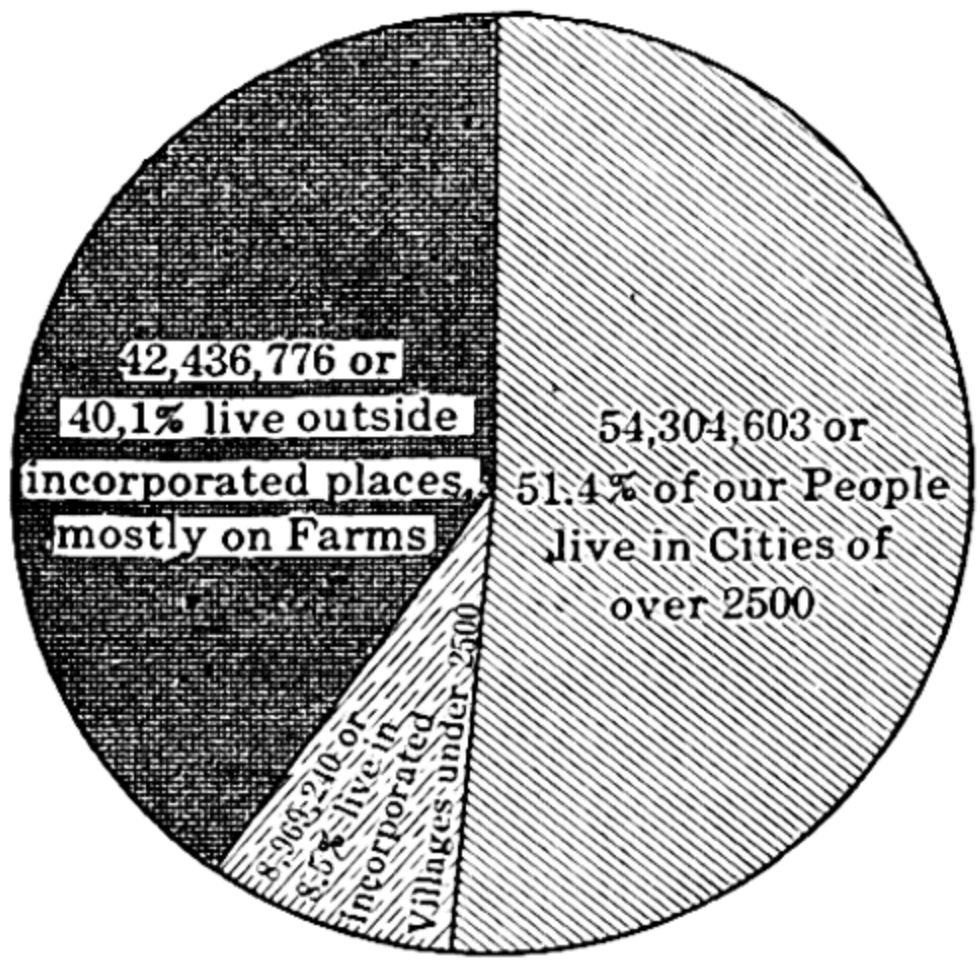


FIG. 69

Determine whether or not these statements are borne out by the graph.

5. In connection with the graph shown in Fig. 70 the following statement is made:

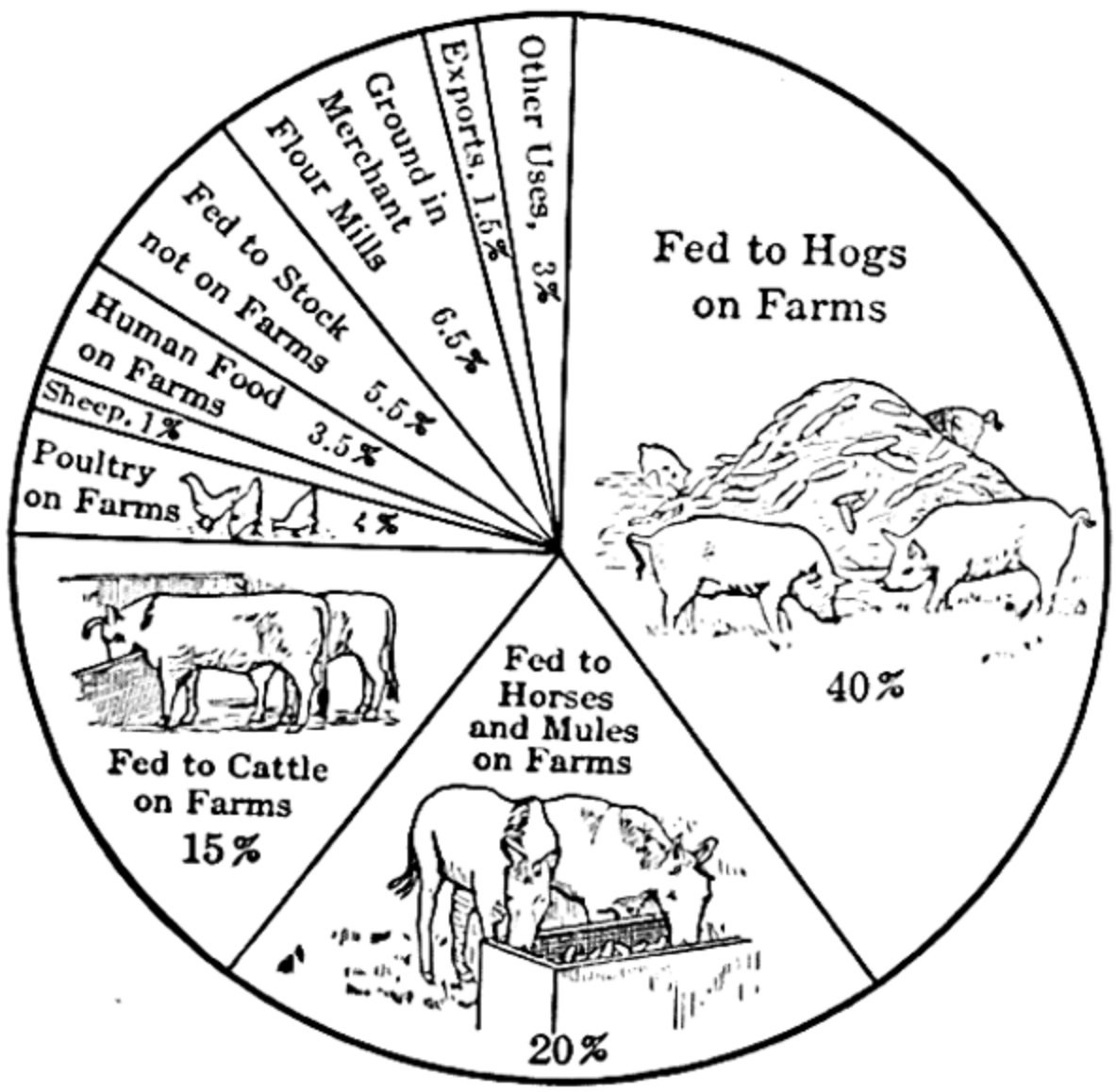


FIG. 70

"The uses of corn harvested for grain in the United States, based on estimates by the U. S. Department of Agriculture. More than 85 per cent is fed to live stock and somewhat less than 10 per cent is used directly for food."

Determine whether or not these statements are borne out by the graph.

QUESTIONS FOR REVIEW

1. Which state of the United States is most nearly a square? Measure its length and width on the map in your geography to the nearest tenth of an inch. Note the scale used on the map and write these dimensions in miles. Compute the area of this state in square miles. Where can you find a check for your answer?

2. Select another state of the United States which is most nearly a rectangle and proceed as in Exercise 1. Is your answer right?

3. In finding the area of each of the following rectangles, tell what you must do before making the computation. Find the area of each rectangle in terms of the unit of area shown.

(*a*) 13 yd. 6 in. by 12 ft. (? sq. ft.)

(*b*) 12 yd. 1.5 ft. by 3 yd. 9 in. (? sq. yd.)

(*c*) 36 rd. by 58.4 yd. (? sq. yd.)

(*d*) 154 rd. by 154 yd. (? sq. rd.)

(*e*) 864 rd. by 847 yd. (? acres)

4. Each of the following rectangles has a perimeter of one mile. Find out which has the greatest area:

(*a*) a rectangle twice as long as wide.

(*b*) a rectangle three times as long as wide.

(*c*) a rectangle four times as long as wide.

5. Draw the plan of a triangle whose sides are 20′, 16′, and 12′, respectively. Select your own scale in doing this.

(*a*) What kind of a triangle does this seem to be?

(*b*) What are the values of its angles?

(*c*) What is the area of the original triangle?

6. The side lines of a certain house lot are parallel, being 110′ and 106′ respectively. The front line makes right angles with each side line. The front line is 50.3′ and the back line is 60.7′. Find the perimeter and area of this lot. Draw a plan of it.

CHAPTER VII

PERCENTAGE APPLIED TO PROBLEMS OF BUSINESS

I. Review of Per Cent. Finding Per Cents of Numbers.

67. In Chapter IV you learned that *per cent* means "*rate per hundred*," "*number per hundred*," or "*number of hundredths.*"

In Chapters IV, V, and VI you learned to apply the idea of per cent to numbers, to some problems of buying and selling, and to measures. In this chapter it is used in various problems of business with which every person should be familiar.

The idea of per cent is not as old as the other number ideas you have had in this book. The Babylonian schoolboy did not learn to solve problems in percentage on his clay tablet. The Egyptian, Greek, Roman, and Hindu schoolboys did not know that any such thing existed. Instead of 25%, $33\frac{1}{3}\%$, etc., they used the common fractions, $\frac{1}{4}$, $\frac{1}{3}$, etc. It was the sons of the merchants of Italy, probably not more than 400 years ago, who first studied percentage in school. Percentage with its applications to problems of business like discount and interest grew out of man's need for it in the extensive trading between the Italian cities of the 16th century and the other cities on the shores of the Mediterranean Sea.

Mankind's need for them has caused the development of all the processes you have learned in this book. These things are *used* by people day after day. You should strive to make yourself as proficient in their use as possible.

It would be well for you to review the third part of Chapter IV which deals with percentage, giving especial attention to the Table of Per Cents on page 91, which expresses the most used per cents as fractional ratios reduced to lowest terms. The following exercises are planned to help you in the review and will give you material for drill.

REVIEW EXERCISES

Answer as many as possible of these exercises without using pencil or paper. In the more difficult ones you may need to use the equation as on page 90.

1. What per cent of 50 is 20? 15? 16? 12? 8? 18?
2. What per cent of 25 is 20? 10? 15? 5? 8? 12?
3. What per cent of 64 is 24? 48? 56? 40? 8? 4?
4. What per cent of 36 is 9? 12? 24? 18? 30? 36?
5. What per cent of 72 is 24? 36? 48? 18? 60? 72?
6. 24 is what per cent of 60? Of 96? Of 24? Of 32?
7. 28 is what per cent of 35? Of 56? Of 42? Of 28?
8. 5 is what per cent of 6? Of 8? Of 10? Of 15? Of 3?
9. 8 is what per cent of 10? Of 20? Of 6? Of 16? Of 5?
10. 30 is what per cent of 40? Of 25? Of 24? Of 20? Of 80?
11. 18 books are what per cent of 30 books? Of 48 books?
12. What per cent of 8 feet is 48 inches? 60 inches? 7 feet?
13. What per cent of 16 feet is 4 yards? 2 yards?
14. $1.20 is what per cent of 80 cents? Of $1.00?

15. What per cent of $7.50 is $5.00? $3.00? $6.00?

16. What per cent of a gallon is 2 pints? 3 pints? 4 pints? 6 pints? 8 pints? 12 pints?

17. What per cent of a yard is 1 foot? 2 feet 3 inches? 2 feet 6 inches? 3 feet 9 inches? 6 feet?

18. What per cent of a ton is 750 pounds? 1200 pounds? 1800 pounds? 2000 pounds? 3000 pounds?

19. What per cent of a square yard is 144 square inches? 5 square feet? 12 square feet? 18 square feet?

20. What per cent of a mile is 880 yards? 220 yards? 440 yards? 100 yards? 1000 yards?

21. Of 125 grains of wheat planted 90 sprouted. What per cent sprouted?

22. Of 300 grains of oats planted 261 sprouted. What per cent sprouted?

23. Of 200 kernels of corn planted 165 sprouted. What per cent sprouted?

24. A farmer set out 240 apple trees; 36 of them died. What per cent of the trees died?

25. A farmer set out 175 peach trees; 42 of them died. What per cent of the trees lived?

26. A farmer set out 75 quince trees; 15 of them died. What per cent of the trees lived?

27. In September flour was selling at $9.00 per barrel; in November flour was selling at $12 per barrel. What was the per cent of increase?

28. In September potatoes were selling at $1.40 per bushel; in November at $2.00 per bushel. What was the per cent of increase?

29. In August coal was retailing at $15 per ton; in November it was retailing at $16.50 per ton. What was the per cent of increase?

30. In 1922 a farmer paid his hired man \$300 for a year's work; in 1923 he paid him \$350 for a year's work. What was the per cent of increase?

In Exercises 31–35 compute the per cents correct to the nearest *tenth* of one per cent as shown in the solution of the example that follows.

31. The price of an automobile was reduced from \$490 to \$360. What was the per cent of reduction?

SOLUTION.

\$490
360
\$130 (reduction)

.2653
490)130.0000
980
3200
2940
2600
2450
1500
1470

NOTE: To get the answer correct to the nearest *tenth* of one per cent, you must carry the division far enough to obtain the figure in the fourth decimal place. If this figure is 5 or more, increase the one preceding it by 1 before you write the per cent; drop any figure in the fourth decimal place which is less than 5.

Ans. 26.5%

32. The price of an automobile was increased from \$1280 to \$1350. What was the per cent of increase?

33. The price of an automobile was increased from \$1190 to \$1280. What was the per cent of increase?

34. The price of an automobile was reduced from \$750 to \$695. What was the per cent of reduction?

35. The cost of an automobile was \$1950. After being used one year, it was sold for \$1500. What was the per cent of loss?

68. Per Cents—Fractions. These exercises are for oral work. Give either the equivalent fraction or the equivalent per cent as required.

$50\% = ?$	$\frac{3}{4} = ?\,\%$	$\frac{4}{4} = ?\,\%$
$33\frac{1}{3}\% = ?$	$\frac{7}{10} = ?\,\%$	$110\% = ?$
$10\% = ?$	$\frac{1}{8} = ?\,\%$	$\frac{4}{3} = ?\,\%$
$25\% = ?$	$\frac{2}{3} = ?\,\%$	$100\% = ?$
$16\frac{2}{3}\% = ?$	$\frac{3}{8} = ?\,\%$	$175\% = ?$
$20\% = ?$	$\frac{4}{5} = ?\,\%$	$\frac{9}{8} = ?\,\%$
$12\frac{1}{2}\% = ?$	$\frac{1}{6} = ?\,\%$	$\frac{9}{4} = ?\,\%$
$66\frac{2}{3}\% = ?$	$\frac{5}{6} = ?\,\%$	$166\frac{2}{3}\% = ?$
$37\frac{1}{2}\% = ?$	$\frac{5}{8} = ?\,\%$	$250\% = ?$
$75\% = ?$	$\frac{9}{10} = ?\,\%$	$\frac{7}{6} = ?\,\%$
$90\% = ?$	$\frac{2}{5} = ?\,\%$	$\frac{11}{8} = ?\,\%$
$87\frac{1}{2}\% = ?$	$\frac{1}{20} = ?\,\%$	$225\% = ?$
$83\frac{1}{3}\% = ?$	$\frac{7}{8} = ?\,\%$	$500\% = ?$
$62\frac{1}{2}\% = ?$	$\frac{3}{10} = ?\,\%$	$\frac{13}{8} = ?\,\%$
$5\% = ?$	$\frac{1}{25} = ?\,\%$	$187\frac{1}{2}\% = ?$
$8\frac{1}{3}\% = ?$	$\frac{1}{16} = ?\,\%$	$\frac{1}{100} = ?\,\%$
$11\frac{1}{9}\% = ?$	$\frac{1}{7} = ?\,\%$	$\frac{1}{200} = ?\,\%$
$6\frac{1}{4}\% = ?$	$\frac{1}{50} = ?\,\%$	$\frac{1}{300} = ?\,\%$
$14\frac{2}{7}\% = ?$	$\frac{1}{9} = ?\,\%$	$\frac{1}{2}\% = ?$
$2\% = ?$	$\frac{1}{12} = ?\,\%$	$\frac{1}{10}\% = ?$

69. Per Cents Expressed as Decimals.—These exercises are for written work. Write these per cents as decimals.

1. 25%	**5.** 1.25%	**9.** 80%	**13.** 2%
2. 75%	**6.** 10%	**10.** 1.60%	**14.** 6%
3. 38%	**7.** 40%	**11.** 5%	**15.** 1.09%
4. 42%	**8.** 70%	**12.** 8%	**16.** $12\frac{1}{2}\%$

17. $87\frac{1}{2}\%$	21. $24\frac{3}{4}\%$	25. $\frac{1}{4}\%$	29. $6\frac{1}{3}\%$
18. $37\frac{1}{2}\%$	22. $6\frac{1}{4}\%$	26. $33\frac{1}{3}\%$	30. $133\frac{1}{3}\%$
19. $4\frac{1}{2}\%$	23. $3\frac{3}{4}\%$	27. $66\frac{2}{3}\%$	31. 125%
20. $\frac{1}{2}\%$	24. $30\frac{1}{4}\%$	28. $83\frac{1}{3}\%$	32. $166\frac{2}{3}\%$

70. Finding Per Cents of Numbers. The processes used are illustrated in the following examples.

EXAMPLE 1. On a spelling test of 20 words:

John missed 10%. How many words did he miss?

Mary missed 20%. How many words did she miss?

Henry missed 25%. How many words did he miss?

James missed 30%. How many words did he miss?

SOLUTIONS. John missed 10%, or $\frac{1}{10}$, of 20 words.

$\frac{1}{10}$ of $20 = 2$.

Ans. John missed 2 words.

Mary missed 20%, or $\frac{1}{5}$, of 20 words. $\frac{1}{5}$ of $20 = 4$.

Ans. Mary missed 4 words.

EXAMPLE 2. On a spelling test of 50 words:

James spelled correctly 94%. How many words did he spell correctly?

Mary spelled correctly 90%. How many words did she spell correctly?

Henry spelled correctly 84%. How many words did he spell correctly?

John spelled correctly 80%. How many words did he spell correctly?

Jennie spelled 35 words correctly. What per cent did she spell correctly? What per cent did she misspell?

SOLUTION. James spelled correctly 94% or .94 of 50 words.

$$.94 \times 50 = 47.00.$$

Ans. James spelled correctly 47 words.

EXERCISES

1. Find 20% of 2240 pounds.
2. Find $12\frac{1}{2}\%$ of 128 cubic feet.
3. Find $37\frac{1}{2}\%$ of 160 square rods.
4. Find 75% of 5280 feet.
5. Find 90% of 320 rods.
6. Find $87\frac{1}{2}\%$ of 1728 cubic inches.
7. Find $62\frac{1}{2}\%$ of 144 square inches.
8. Find $66\frac{2}{3}\%$ of 1728 cubic inches.
9. Find $66\frac{2}{3}\%$ of 231 cubic inches.
10. Find $62\frac{1}{2}\%$ of 2000 pounds.
11. Find $37\frac{1}{2}\%$ of 2150.42 cubic inches.
12. Find 80% of 640 acres.
13. Find $66\frac{2}{3}\%$ of 135 cubic inches.
14. Find 5% of \$874.20.
15. Find 6% of \$364.
16. Find $5\frac{1}{2}\%$ of \$152.50.
17. Find $112\frac{1}{2}\%$ of \$864.
18. Find 225% of \$1486.
19. Find $2\frac{1}{2}\%$ of \$3564.
20. Find $1\frac{1}{4}\%$ of 5280 feet.
21. Find 8.2% of 1728 cubic inches.
22. Find 11.3% of 2150.42 cubic inches.
23. Find 8.1% of \$756.42.
24. Find $\frac{1}{4}\%$ of 3,370,000.
25. Find $\frac{1}{8}\%$ of 670,000.
26. Find 0.2% of 5,000,000.
27. Find 0.5% of \$200.
28. Find $\frac{1}{2}\%$ of 5280 feet.
29. Find $4\frac{1}{2}\%$ of \$1080.
30. Find 0.8% of 125,000.

Try starting at different places in the following table and write as many answers as you can in *three* minutes under the direction of the teacher, who will keep the time. Keep the following records of successive tests: (1) number attempted, (2) number right, and (3) the per cent that the number right is of the number attempted for each test. Draw the three graphs. Strive to make the records so that the first two graphs will climb higher and higher and the third will reach 100 and stay there. The class champion is the one whose per cent remains at 100 while each succeeding test shows more problems done. If more than one pupil has a per cent of 100, the one having most problems done in three minutes is champion. When the per cent drops below 100 the championship is lost. Team championships may be determined by taking team averages. This table and another like it in Chapter VIII may be used repeatedly as you go on through the applications of percentage.

TABLE IX

$12\frac{1}{2}\%$ of 32	$33\frac{1}{3}\%$ of 96	70 % of 90
$66\frac{2}{3}\%$ of 66	30 % of 20	$37\frac{1}{2}\%$ of 56
60 % of 55	$62\frac{1}{2}\%$ of 40	$83\frac{1}{3}\%$ of 18
25 % of 48	10 % of 80	80 % of 50
$16\frac{2}{3}\%$ of 54	75 % of 36	$16\frac{2}{3}\%$ of 78
40 % of 75	$87\frac{1}{2}\%$ of 16	90 % of 30
$37\frac{1}{2}\%$ of 64	40 % of 45	$87\frac{1}{2}\%$ of 24
$33\frac{1}{3}\%$ of 72	$83\frac{1}{3}\%$ of 48	75 % of 44
90 % of 50	70 % of 60	10 % of 28
50 % of 78	60 % of 35	$62\frac{1}{2}\%$ of 32
$83\frac{1}{3}\%$ of 60	$66\frac{2}{3}\%$ of 42	30 % of 70
20 % of 65	$37\frac{1}{2}\%$ of 88	$16\frac{2}{3}\%$ of 54

$66\frac{2}{3}\%$ of 45	30 % of 50	$37\frac{1}{2}\%$ of 72
40 % of 35	$37\frac{1}{2}\%$ of 40	50 % of 46
$12\frac{1}{2}\%$ of 48	$62\frac{1}{2}\%$ of 48	$83\frac{1}{3}\%$ of 42
75 % of 32	$66\frac{2}{3}\%$ of 75	25 % of 76
$33\frac{1}{3}\%$ of 27	60 % of 15	10 % of 62
20 % of 90	$16\frac{2}{3}\%$ of 84	$87\frac{1}{2}\%$ of 88
$87\frac{1}{2}\%$ of 800	75 % of 840	$16\frac{2}{3}\%$ of 660
10 % of 240	20 % of 950	60 % of 150
25 % of 340	$33\frac{1}{3}\%$ of 870	$37\frac{1}{2}\%$ of 320
$83\frac{1}{3}\%$ of 120	$12\frac{1}{2}\%$ of 400	70 % of 250
50 % of 900	40 % of 150	$66\frac{2}{3}\%$ of 270
$37\frac{1}{2}\%$ of 560	$66\frac{2}{3}\%$ of 150	$62\frac{1}{2}\%$ of 480

Table X

Use like Table IX, with a time limit of *four* minutes.

5 % of \$12.00	20 % of \$22.25	4 % of \$17.50
$12\frac{1}{2}\%$ of 28.40	$16\frac{2}{3}\%$ of 12.60	$33\frac{1}{3}\%$ of 24.60
3 % of 11.00	5 % of 17.00	10 % of 23.50
$16\frac{2}{3}\%$ of 34.20	$37\frac{1}{2}\%$ of 24.00	2 % of 27.50
20 % of 13.75	4 % of 12.50	$2\frac{1}{2}\%$ of 10.00
$2\frac{1}{2}\%$ of 8.00	10 % of 11.50	$37\frac{1}{2}\%$ of 48.00
10 % of 17.50	6 % of 15.00	20 % of 18.50
4 % of 22.50	$2\frac{1}{2}\%$ of 12.00	$16\frac{2}{3}\%$ of 16.20
$33\frac{1}{3}\%$ of \$18.60	$12\frac{1}{2}\%$ of \$24.80	30 % of \$16.50
2 % of 12.50	3 % of 17.00	$12\frac{1}{2}\%$ of 32.80
$37\frac{1}{2}\%$ of 32.00	$33\frac{1}{3}\%$ of 15.30	3 % of 21.00
30 % of 18.50	8 % of 12.50	5 % of 18.40
6 % of 12.00	40 % of 13.50	40 % of 19.50
15 % of 22.00	15 % of 26.00	8 % of 17.50
40 % of 15.50	30 % of 14.00	15 % of 16.00
8 % of 25.50	2 % of 17.50	6 % of 17.50

APPLIED PROBLEMS

1. A farmer had 375 sheep and sold 20% of them. How many did he sell?

2. In a school of 530 students 40% are boys. How many boys are there in the school?

3. A grain dealer bought 2480 barrels of flour and sold $12\frac{1}{2}\%$ of them. How many barrels did he sell?

4. A man's income is \$1200. He saves $16\frac{2}{3}\%$ of it. How much does he save?

5. A gasoline tank contained 260 gallons; 15% of it was sold. How many gallons were sold?

6. In July, 1923, a reduction in the price of gasoline from 26.5 cents per gallon to 15.5 cents was brought about in the state of South Dakota. This reduction was reported in the papers as approximately 40%. Was this report correct?

7. An automobile cost \$1800. Since it was purchased, it has decreased $16\frac{2}{3}\%$ in value. How much has it decreased in value? What is it worth now?

8. A farmer had 350 acres of land and sold 20% of it. How many acres did he sell? How many acres remained?

9. A man earns \$1500 in a year. He spends 90% of it. How many dollars does he spend? How much has he left?

10. In a certain lumber mill 34,497 board feet are sawed daily Of this 5% is trimmed off. How many board feet are trimmed off?

11. In another mill 18,765 board feet are sawed daily; 9% is trimmed off. How many board feet are trimmed off?

12. An automobile selling for \$1395 is reduced 25%. Find the price after the reduction.

13. A's salary per year has been \$1780. He is given an increase in salary of 15%. What is his salary after the increase has been given him?

14. An automobile which costs \$1095 is offered for sale at a reduction of 15%. At what price is it offered?

15. In April, 1923, the wages of carpenters in a certain city were \$49.50 per week of 45 hours. In May of that year this scale was increased 10%. Determine the wages per hour after the increase.

16. A mechanic who received \$21.00 per week in 1913 was getting 215% of this amount in 1922 and 250% of this amount in 1923. What were his wages per week in 1922? In 1923?

17. A proportionate tax on salaries is one where the per cent remains the same regardless of the amount of money involved. The following salaries are taxed at the rate of 5%:

(1) \$2500, (2) \$25,000, (3) \$60,000. How much tax will be paid on each of these salaries?

18. A progressive tax is one where the rate increases as the amount involved increases. If the rates on the salaries in the preceding exercise were as given here, determine how much tax would be paid on each salary.

On (1), rate 4%; on (2), rate 8%; on (3), rate 15%.

19. Approximately \$2,519,000,000 was invested in the building of new homes, apartment houses, and business structures in the largest cities of the United States in 1922. Approximately \$1,627,000,000 went into residential construction or more than 60% of the year's total investment in new structures. Determine whether this statement is approximately correct.

II. Discount

71. A *discount* is a reduction made from a given price. This reduction is generally stated as so many per cent of the given price, which is called the *list price* or *marked price.*

The price after the discount has been taken off is called the *net price.*

APPLIED PROBLEMS

1. A teacher who orders books amounting to $36.50 gets a discount of $16\frac{2}{3}\%$. What is the net price of these books?

Solution. List price. $36.50

Less $16\frac{2}{3}\%$, or $\frac{1}{6}$, 6.08

$30.42 (Net price of books)

Ans. $30.42.

2. A dozen copies of a book are ordered. The list price of the book is $1 25 The discount is 20%. Find the net cost of the 12 books.

3. A hammock marked to sell at $15.60 is reduced 20%. Find the net price.

4. The marked price of a piano is $425. A discount of 15% is allowed for cash. Find the net cost.

5. A farmer buys a reaper at $85 less a discount of 5% for cash. What does he pay in cash for the reaper?

6. A Victrola is sold for $200, subject to a discount of $12\frac{1}{2}\%$ for cash. What is the net cost of the Victrola if cash is paid?

7. The following are advertised at a reduction of $33\frac{1}{3}\%$ on these prices: $3.50 satin; $4.00 crêpe; $1.00 pongee; $3.50 silk. Find the net price of each.

8. A contractor orders 10 kegs of nails. There are 50 lbs. of nails in each keg. The price per pound is $4\frac{1}{2}$ cents less a discount of 15%. Find the net cost.

9. A farmer buys a corn harvester for \$150 less a discount of $12\frac{1}{2}\%$ for cash. How much money will he need if he pays cash for the harvester?

10. Oriental rugs are advertised for the August sale at discounts of 25%. Find the net price of each of the following: rug *A* listed at \$179; rug *B* listed at \$225; rug *C* listed at \$95; rug *D* listed at \$345.

11. An automobile listed at \$1595 is offered for quick sale at a discount of 15%. Find the net price asked.

12. Determine whether the suits in the following advertisement of a reputable clothing store have been priced as advertised:

ONE THIRD OFF
EVERY 2 PIECE
SUMMER SUIT IN OUR STORE

\$15.00	Two-Piece Suits	Sale Price	\$10.00
\$17.50	Two-Piece Suits	Sale Price	\$11.67
\$20.00	Two-Piece Suits	Sale Price	\$13.34
\$25.00	Two-Piece Suits	Sale Price	\$16.67
\$30.00	Two-Piece Suits	Sale Price	\$20.00

13. This is also an advertisement of a reputable clothing store. Determine whether the revised prices here are correct to the nearest dollar.

S-U-I-T-S

at $\frac{1}{3}$ off

\$40 Suits	- - - - -	Now \$27
\$50 Suits	- - - - -	Now \$34
\$60 Suits	- - - - -	Now \$40
\$75 Suits	- - - - -	Now \$50

14. Determine the per cent reduction on the suits in this sale.

MARK-DOWN SALE

$25 Suits $19.50
$30 Suits $24.50
$35 Suits $29.50
$40 Suits $33.50
$45 Suits $37.50
$50 Suits $42.50

15. Determine the amount of the discount and the net price on each article in the following list:

Article	Former Price	Per Cent of Discount	Discount	Net Price
a. Overcoat	$20.00	15 %	?	?
b. Overcoat	25.00	18 %	?	?
c. Overcoat	30.00	$16\frac{2}{3}$ %	?	?
d. Overcoat	50.00	15 %	?	?
e. Overcoat	65.00	$16\frac{2}{3}$ %	?	?
f. Sideboard	105.00	15 %	?	?
g. Sideboard	275.00	40 %	?	?
h. Sideboard	350.00	35 %	?	?
i. Sideboard	265.00	30 %	?	?
j. Bureau	250.00	48 %	?	?
k. Bureau	165.00	52 %	?	?
l. Table	185.00	$12\frac{1}{2}$ %	?	?
m. Table	205.00	54 %	?	?
n. Table	250.00	$37\frac{1}{2}$ %	?	?
o. Table	320.00	35 %	?	?

III. Commission

72. ***Commission*** is the money paid to anyone who transacts business for you. It is generally a percentage of the amount of money used for a sale or purchase.

Thus a real estate agent who sells a house and lot for you may charge 5% of the selling price as his *commission* for doing the business.

If a commission is charged against the purchaser, the commission must be added to the price to obtain the total cost. If a commission is charged against the seller, it must be subtracted from the price to obtain the net price.

Example 1. A house and lot is sold for $5400 by a real estate agent at a commission of 5%. Find his commission. Find the net receipts of this sale.

Solution.

$5400	$5400
.05	270
$270.00	$5130

Ans. $270 commission; $5130 net receipts of sale.

Example 2. A traveling salesman sells for his firm goods to the amount of $16,500 in one year. His commission is $12\frac{1}{2}\%$ of his sales. Find his commission. Find the net receipts for these goods.

Solution. $12\frac{1}{2}\% = \frac{1}{8}$.

$\frac{1}{8}$ of $16,500 = $2,062.50.

$16,500
2,062.50
$14,437.50

Ans. $2,062.50 commission; $14,437.50 net receipts.

APPLIED PROBLEMS

1. Tom is allowed a commission of 20% of all the money received from the sale of eggs which he delivers to customers. His record of sales for one week, which he submitted to his father, was as follows:

Monday, 5 doz. @ 48¢	.....
Tuesday, 3 doz. @ 50¢	.....
Wednesday, 4 doz. @ 50¢	.....
Thursday, 6 doz. @ 45¢	.....
Friday, 8 doz. @ 45¢	.....
Saturday, 10 doz. @ 46¢	.....
Total.....	

Carry out the amount received for each day; find the total receipts and the amount of Tom's commission for the week.

2. Harry sells *Saturday Evening Posts* to earn his spending money and receives a commission of 40% of the amount of his sales. His record for four successive weeks was as follows:

First week, 18 @ 5¢	.....
Second week, 26 @ 5¢	.....
Third week, 25 @ 5¢	.....
Fourth week, 17 @ 5¢	.....
Total.....	

Find the total amount of his sales and the total amount of his commssion from these sales.

3. On an automobile selling at $1095 an agent's commission is 20%. In one month he sells 12 cars. Find his commission.

4. A house lot containing 8750 square feet of land is sold by an agent at 23 cents per square foot. His com-

mission is 2% of the selling price. Find his commission and the net receipts of the sale.

5. A coal company gives a real estate agent a 5% commission on all orders for coal taken through his office. During the month he takes orders for 82 tons at $15 per ton. Find his commission.

6. A clerk in a store receives a 10% commission on all orders given by people introduced by him. During a month such orders amount to $325. What is his commission?

7. Mr. Jones is agent for Victrolas. He receives a 20% commission on all his sales. He sells in one month 2 Victrolas at $25 each; 3 at $50 each; 1 at $175. What is his commission for the month?

8. An agent who sells farm machinery gets a commission of 15% on his sales. During the season he sells the following: 3 harrows at $38.50 each; 5 mowers at $48 each; 3 hayrakes at $35 each; 2 reapers at $85 each. Find his commission.

9. A book agent's commission is $37\frac{1}{2}\%$ on his sales. During the week he sells 28 books at $2.50 each. How much is his commission. How much money should he send the company?

10. An agent in New York sells a carload of hay for a farmer. The hay is sold for $272.50 at a commission of 8%. Find the commission and the amount of money the agent should send the farmer.

11. A dealer in Portland, Me., orders his agent in Chicago to buy him a carload of horses. The agent buys 28 horses at an average price of $172.50 each. His rate of commission is 5%. What is his commission? What is the total cost of the carload of horses?

12. A farm agency sells a farm for Mr. Brown for $4300 at a commission of 8%. How much does Mr. Brown receive for the farm?

13. A grain dealer buys through his agent in Buffalo the following. Determine the agent's commission on each transaction and the gross cost in each case if the agent's rate of commission is $2\frac{1}{2}\%$. (To get the gross cost, add the agent's commission to the amount involved in the purchase.)

Article and Quantity	Price for Each	Commission	Gross Cost
a. 225 barrels Winter patents	$8.75	?	?
b. 175 barrels Winter patents	9.75	?	?
c. 150 barrels Winter patents	8.85	?	?
d. 320 barrels Spring patents	8.40	?	?
e. 275 barrels Spring patents	8.20	?	?
f. 125 barrels Spring patents	8.15	?	?
g. 850 bushels wheat	1.24	?	?
h. 1200 bushels wheat	1.18	?	?
i. 1800 bushels wheat	1.22	?	?
j. 1200 bushels corn	.77	?	?
k. 1560 bushels corn	.57	?	?
l. 850 bushels corn	.62	?	?
m. 1450 bushels oats	.51	?	?
n. 950 bushels oats	.42	?	?
o. 1120 bushels oats	.34	?	?
p. 550 bushels barley	.92	?	?
q. 880 bushels barley	.58	?	?
r. 950 bushels rye	1.27	?	?
s. 1150 bushels rye	.86	?	?
t. 3000 pounds clover seed (cwt.)	18.32	?	?
u. 5000 pounds clover seed (cwt.)	18.50	?	?
v. 8000 pounds timothy seed (cwt.)	8.33	?	?
w. 12,000 pounds timothy seed (cwt.)	8.82	?	?

IV. Interest

73. ***Interest*** is money paid for the use of money.

The ***principal*** is the sum of money loaned or borrowed.

The ***rate of interest*** is the per cent of the principal paid per year.

The ***amount*** is the sum of the principal and interest for the given time.

Example 1. A lends B \$500 at 5% for 1 year. How much will B owe A at the end of the year?

Solution. The *principal* is \$500. The *rate of interest* is 5%. The *interest* is 5% of \$500, or \$25. The *amount* due A at the end of the year is \$500 + \$25 = \$525.

Example 2. Mr. J. Smith borrows \$2500 from the bank for 60 days, interest at $5\frac{1}{2}\%$. How much interest must he pay? (360 days are usually taken as the commercial year, and 30 days as the commercial month.)

Solution. 60 days $= \frac{60}{360}$ yr. $= \frac{1}{6}$ yr. The interest for one year is $\$2500 \times .05\frac{1}{2} = \$137.50$.
Hence the interest for $\frac{1}{6}$ yr. is

$$\frac{1}{6} \text{ of } \$137.50 = \$22.92.$$

Summary. (1) *Multiply the principal by the given rate of interest.* (For example, $\$2500 \times .05\frac{1}{2} = \$137.50$.) *This product is the interest for one year.*

(2) *Express the time as a fractional part of a year, either by writing the number of months given over 12, or the number of days given over 360.* (For example, 3 months = $\frac{3}{12}$ year; 60 days = $\frac{60}{360}$ year.)

(3) *Multiply the interest found for one year by this fraction.* (For example, $\$137.50 \times \frac{1}{6} = \$22.92$.) *This product is the interest for the given time at the given rate.*

PREPARATORY EXERCISES

In these exercises you should first find the interest for one year and then find that fractional part of this interest which the time is of one year. Give answers to the nearest cent.

1. What is the interest on \$100 at 6% for one year? For 6 months? For 4 months? For 3 months? For 2 months? For 1 month?

2. What is the interest on \$100 at 4% for one year? For 6 months? For 3 months? For 4 months? For 2 months? For 1 month?

3. What is the interest on \$100 at 5% for one year? For 6 months? For 3 months? For 4 months? For 60 days (2 months)?

4. What is the interest on \$100 at 7% for one year? For 6 months? For 3 months? For 4 months? For 2 months? For 30 days?

5. What is the interest on \$100 at $4\frac{1}{2}\%$ for one year? For 6 months? For 4 months? For 3 months? For 60 days? For 30 days?

The following data may be used, as in Exercises 1–5, for further practice in computing interest without the use of paper and pencil.

Principal: \$200; \$50; \$10; \$400; \$500; \$20; \$5; \$1.
Rate per cent: 6%; 7%; 5%; 3%; 8%; $4\frac{1}{2}\%$; $5\frac{1}{2}\%$.
Time: 6 mo.; 4 mo.; 3 mo.; 1 mo.; 60 da.; 30 da.

EXERCISES

Compute the interest on:

1. \$550 at 5% for 1 year; for 6 months.
2. \$4285 at 4% for 60 days; for 40 days.
3. \$650 for 6 months at 6%; for 2 months.

4. \$1098 for 7 months at 6%; for 5 months.
5. \$1476 at $4\frac{1}{2}\%$ for 9 months; for 3 months.

NOTE. Where fractions appear cancellation may be used.

Thus, in Exercise 5, the interest for 9 months is

$$\overset{123}{\cancel{1476}} \times \frac{9}{200} \times \frac{9}{\cancel{12}} = \frac{9963}{200} = 49.82$$

6. \$1764 at $5\frac{1}{2}\%$ for 8 months.
7. \$2908 at $4\frac{1}{2}\%$ for 1 year, 2 months ($\frac{14}{12}$ yr.).
8. \$3052 at 5% for 3 months, 15 days ($\frac{105}{360}$ yr.).
9. \$2592 at $4\frac{1}{2}\%$ for 2 months, 12 days.
10. \$2880 at 4% for 6 months, 20 days.
11. \$3204 at $5\frac{1}{2}\%$ for 4 months, 24 days.
12. \$4068 at 6% for 2 months, 3 days.
13. \$6250 at 5% for 3 months, 8 days.

Compute the interest and find the amount due on:

14. \$650 at 6% for 3 months; for 5 months.
15. \$1275 at 5% for 5 months; for 7 months.
16. \$1485 at 5% for 10 months; for 6 months.
17. \$2250 at $5\frac{1}{2}\%$ for 8 months, 8 days.
18. \$2750 at $5\frac{1}{2}\%$ for 6 months, 12 days.
19. \$8788 at 4% for 3 months; for 2 months.
20. \$85.60 at 4% for 4 months; for 5 months.
21. \$3880 at $5\frac{1}{2}\%$ for 7 months: for 6 months.
22. \$2960 at 5% for 5 months; for 3 months.
23. \$3,000,000 at 4% for 4 months, 12 days.

APPLIED PROBLEMS

1. James has a \$50 Liberty Bond which bears $4\frac{1}{4}\%$ interest and has loaned his father \$75 for which he re-

ceives 6% interest. How much interest does he receive each year from the two investments?

2. A man has $500 loaned at 7% and owns $1000 worth of 4¼% Liberty Bonds. He has a chance to lend $1500 at 5½%. If he can have his $500 loan repaid and can sell his bonds for their full value, will he profit by changing to the single $1500 loan at 5%? How much more interest will he receive each year?

3. A man can invest either all of $2500 at 5½% or half of it at 4% and the other half at 7%. Which investment will pay more interest? How much more will it pay each year?

4. A college has just invested $500,000 new endowment funds. $85,000 is invested at 6%, $150,000 at 5½%, $175,000 at 6½%, and the remainder at 4¼%. What annual income will the college receive from these investments?

5. Compute the interest on $50 for 6 months at 8% and the interest on $50 for 8 months at 6%. If you had $50 and had the choice of the two ways to lend it, which would you choose? Why?

6. Banks sometimes pay at the end of each month to cities and corporations who keep large sums of money on deposit a small rate of interest on the average daily balance for the month.

A certain city treasurer gets 2½% for the first six months of the year and 2¼% for the remainder of the year. The average daily balances are: Jan., $350,000; Feb., $302,400; March, $330,600; April, $275,500; May, $300,300; June, $305,000; July, $252,300; Aug., $234,500; Sept., $360,900; Oct., $375,000; Nov., $390,700; Dec., $95,000. How much interest does the city receive during the year?

74. The Six Per Cent Method of Computing Interest. Draw on the same sheet of cross-section paper the graphs showing the growth of interest on $1, $2, $3, $5 for periods of 1, 2, 3, 4, 5, 6 months at 6%. Compare the interest values for any chosen length of time. You discover that the interest on $2 is always twice the interest on $1 for the same length of time; the interest on $3 is

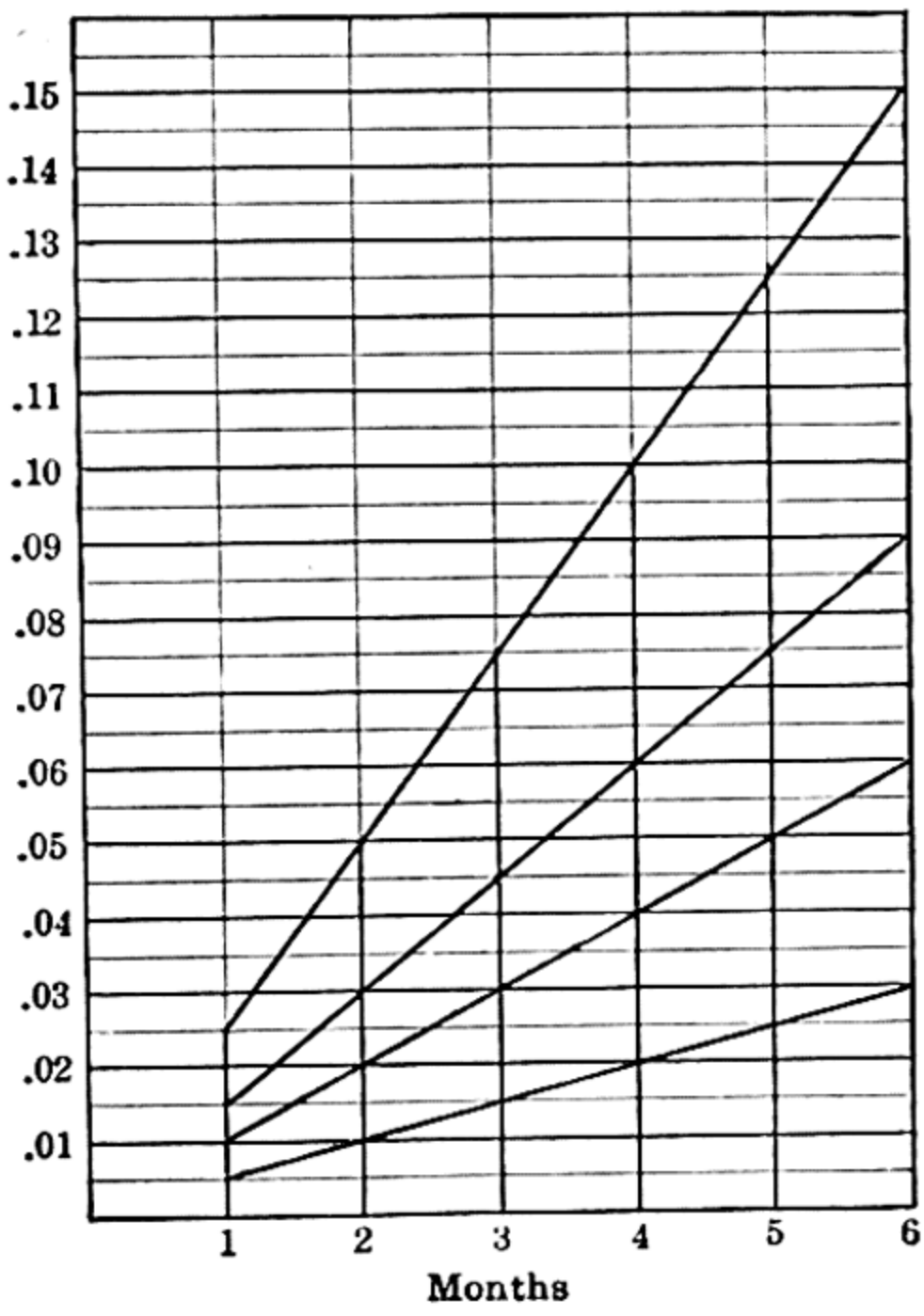


FIG. 71

always three times, and the interest on $5 five times the interest on $1 for the same length of time. You will understand from this why the following rule (in common use) is correct.

THE SIX PER CENT RULE. (For 6% rate):

Find the interest on ONE DOLLAR for the required

length of time at 6%. Multiply this interest by the number of dollars in the principal.

Since the problem by this method begins with the same calculation; namely, the interest on one dollar for the required time, we can make part of that calculation once for all problems, as follows:

Interest on \$1 for 1 yr. at 6% = \$0.06
Interest on \$1 for 1 mo. at 6% = .005
Interest on \$1 for 6 da. at 6% = .001

EXAMPLE. Find the interest on \$350 for 1 year, 3 months, 15 days at 6%.

SOLUTION.

Interest on \$1 for 1 year at 6% is \$0.06.
Interest on \$1 for 3 months at 6% is .015.
Interest on \$1 for 15 days at 6% is .0025.
Total interest on \$1 for required time is \$0.0775.
Interest on \$350 for 1 year, 3 months, 15 days is
$350 \times \$0.0775$, or \$27.13.

CHECK. Use the method of §73 to check your answer.

Ans. \$27.13.

EXERCISES

Use the six per cent method to compute interest on the following, all at the rate of 6%.

1. \$450 for 1 year, 1 month, 12 days.
2. \$700 for 3 months.
3. \$175 for 2 months, 21 days.
4. \$260 for 6 months, 15 days.
5. \$3200 for 60 days.
6. What limitation do you see to this method?

75. You have probably discovered that the rule on the preceding page can be used only when the rate is 6%.

To find the interest at some other rate than 6%:

Multiply the interest computed at the **6%** *rate by the ratio of the required rate to* **6.**

EXAMPLE. Find the interest on $600 for 5 months and 20 days at 7%.

SOLUTION. The interest on $1 for 5 months, 20 days at 6% is 0.028\frac{1}{3}$. $28\frac{1}{3} = \frac{85}{3}$. The ratio of 7% to 6% is $\frac{7}{6}$.

$$\frac{.085}{3} \times \overset{100}{\cancel{600}} \times \frac{7}{\cancel{6}} = 19.83$$

CHECK. Compute the interest by the method of §73.

Ans. $19.83.

EXERCISES

Solve the problems on page 191 by the 6% method.

76. How Interest Grows. The following examples illustrate how interest on money increases with the length of time it is borrowed. Try to answer without help all the questions asked.

EXAMPLE 1. Compute the interest on $100 at 6% for one month; two months; three months; four months; five months; six months.

EXAMPLE 2. Take a sheet of cross-ruled paper and let horizontal distances represent the months and vertical distances the interest values for the different months as in Fig. 72. Draw the graph showing the increase of the interest as time goes on according to the computed values of Example 1.

EXAMPLE 3. Compute the interest on $100 at 6% for $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, and $5\frac{1}{2}$ months. Draw the graph for these values as you did for the values in Example 2, using

the same sheet of paper. Do you get the same line? What kind of line does it seem to be?

EXAMPLE 4. Compare by measurement and by reference to computed values the amount of interest for 2

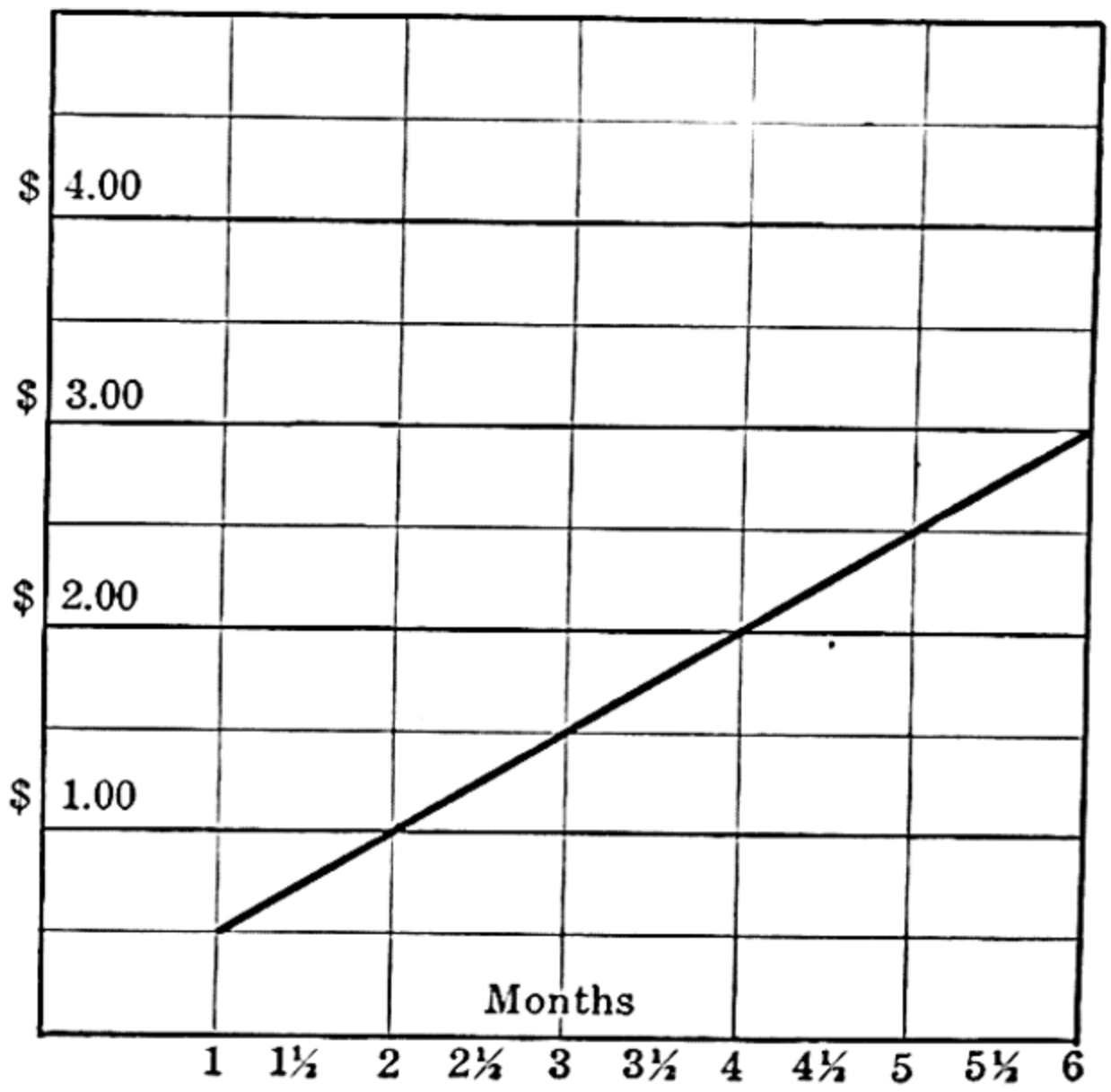


FIG. 72

months with that for 6 months. How does their ratio compare with the ratio of the number of months? Compare also $1\frac{1}{2}$ months and 3 months; $2\frac{1}{2}$ months and 5 months; 3 months and $4\frac{1}{2}$ months.

EXAMPLE 5. If the interest for 2 months is 2 times the interest for one month and for 3 months 3 times the interest for one month (is it?), how could you get quickly the interest for 8 months? 10 months? m months?

EXAMPLE 6. How many months would be required for \$100 at 6% to earn \$5.50 interest?

SOLUTION. Let m = the number of months.

The interest on \$100 at 6% for one month is \$0.50. For m months it would be \$0.50 m.

Then $.50\,m = 5.50$

Divide both sides of the equation by .50:

$$m = 11.$$

CHECK. Interest on \$100 for 1 year at 6% would be $.06 \times 100$, or \$6. For 11 months it would be $\frac{11}{12}$ of \$6, or \$5.50 *Ans.* 11 months.

EXERCISES

1. Draw a graph showing the increase of interest on \$300 at 4% by months for a year.

2. Draw graphs for these two interest problems on the same sheet of paper: (*a*) \$600 at 8%; (*b*) \$800 at 6%. What do you discover? (Compute values for each month for a year. Let each vertical space stand for \$4.)

3. Change your horizontal scale to years and draw graphs showing increase by years of interest on (*a*) \$2000 at 8% for 10 years; (*b*) \$1500 at 6% for 5 years; (*c*) \$1200 at 5% for 9 years. (Let each vertical space stand for \$100.)

4. If the interest on \$100 for one year at 6% is \$6, what is it for 5 years? 8 years? y years?

5. How long will it take \$150 to earn \$45 at 6%? Follow the plan of Example 6 above.

6. How long will it take \$100 at 8% to earn \$100 if the interest accumulates?

7. A Victrola sold on time payments is marked by a music store at \$135. The proprietor will sell it for \$125 cash. The buyer borrows the money at 6%. How many months until the interest paid will equal the cash reduction?

QUESTIONS FOR REVIEW

1. Name the per cent corresponding to each of the following fractional parts: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{2}{5}$, $\frac{5}{6}$, $\frac{1}{4}$, $\frac{7}{8}$, $\frac{3}{5}$, $\frac{1}{3}$, $\frac{3}{8}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{1}{5}$.

2. Is a per cent increase the same as a per cent decrease? Explain by giving illustrations.

3. If $100 had been invested at 6 per cent simple interest on the day of your birth, how much would it amount to now?

4. What is a discount? Why are goods discounted in price? What is the list price? The net price?

5. A man has the choice of buying a $500 bill of goods with a regular 20% discount and an additional 5% discount for cash or with a 25% discount on the assumption that cash is to be paid. Which would be the better for the purchaser?

6. Why might it be worth $60 a year for a man to have the use of $1000? What do you call the $60 he would pay for the use of the money? What rate per cent would it be?

7. What is the difference between commission and profit? Between commission and salary (or wages)?

8. If a man sells an average of 5 books a day during 26 working days in a month and receives a commission of $37\frac{1}{2}\%$ on the list price, which is $3.25 per book, what is his income for the month? Why might he be willing to exchange the job for one paying a salary of $160 per month?

9. A real estate agent in a certain town gets 5% commission on the first $5000 and $2\frac{1}{2}\%$ commission on the remainder of the sale price of a house. What is his commission for selling a house for $8500?

CHAPTER VIII

SUMMARY—MISCELLANEOUS EXERCISES

I. Review Exercises and Problems of Business

1. Mrs. Brown receives a sales slip like Fig. 73 on page 200 with every order the grocer delivers. She adds the items on each slip to check possible errors by the grocer's clerk. Is the addition of the items correct?

2. Mrs. Jones files all her sales slips and adds them at the end of the month to check the monthly statement which the grocer sends. Copy the following itemized accounts, writing each on a small square of paper. Arrange these in order of their dates and add, carrying the total forward from each slip to the next as suggested in Problem 1. The names of the items have been omitted.

July 2	July 3	July 5	July 6	July 7	July 9
.95	.22	.10	.25	.39	.65
1.40	.35	.54	.88	2.15	.98
.09	1.50	.75	3.75	.20	.30
	.40	2.55			1.80

July 11	July 13	July 14	July 16	July 17	July 19
1.25	.23	.25	2.50	.10	.53
.27	2.98	.99	.30	.25	.15
	.50	4.75		.80	1.29
	.75	.18			

July 20	July 21	July 23	July 26	July 28	July 31
.65	1.64	.08	.13	2.85	.77
1.90	.20	1.12	.78		.30
.25	.50	.51			2.64

3. You can probably find a chance at home to check some accounts like Fig. 73. Ask about it. Possibly you can get some of last month's slips to bring to school.

GROCERY & MARKET
STAPLE AND FANCY GROCERIES
Fresh Meats and Vegetables
Phone 2143 and 1837 **96 Ash Avenue**

Springfield, N.Y. 8/7 **19** 23

M Mrs. Brown

No. 416 E. Main St.

Reg. No. ____ **Clerk** 4

	Account Forward		12	42
1	Pk. Potatoes			35
5#	Sugar			47
1	Bread			9
3#	Roast			85
1	Lettuce			15
2	Grape Nuts			40
1	Shredded Wheat			15
1	doz. Oranges			30
1	" Soap (Special)			59
			15	97

FIG. 73

4. Write a series of slips on small squares of paper from dictation by your teacher. See who can get the correct total first, following the plan of Problem 2.

5. Fill out the blanks in the following typical itemized statement of account. Credits for returned goods are to be subtracted.

MC FALL'S

Troy's Greatest Store

Troy, Ohio 7/1/23

Sold to Mr. M. A. Blank
1721 Walton Ave.

			Amount		Credits	
Jun.	2	suit 1.75, skirt 1.95	3	70		
	4	nets .25 thd .05, floss .10, ratine .19		59		
	6	skirt 2.75, dress 7.80, voile .78				
	7	returned dress			7	80
	11	shoes 5.85, socks 2.50, tie .39				
	15	collars.75, silk 5.65, thd. .15				
	18	gingham 1.28, thd. .05, braid .25				
	22	hat 7.25, underwear 3.15, patrn. .15				
	23	returned hat				
	28	Blouse .85, pants 3.98				
		Amount due				

FIG. 74

6. The billing clerk in the same store has these items during the month to enter on Mr. A's account. Make out a statement like the one in Exercise 5 and find the amount due: June 1, linen 4.80, voile 2.64; June 4, hat 1.25, socks .35, tie 1.20; June 9, scrim 1.25, thread .10, braid .16; June 11, returned hat; June 15, velvet 2.65, satin .98, thread .15; June 18, remnant .98, blouse 1.75, buttons .25; June 23, shoes 2.98, cap 1.25, handkerchief .35, coat 7.95.

7. Write from your teacher's dictation the items with dates for another account similar to that of Exercise 6, and find amount due.

8. In the following table: (*a*) Compute the average price, in cents, of a bushel of corn in each state for the years 1919, 1920, and 1921. (Divide the sum of the *three* prices in each line by 3; work out each quotient to *tenths;* then write the answer to the nearest cent. Thus 72.8 cents would be 73 cents; 72.4 cents would be 72 cents.) (*b*) In a similar way compute the average yield in bushels per acre, correct to *tenths* for the years 1919, 1920, and 1921. (As these numbers are given correct to *tenths*, work out each quotient to *hundredths*, and then write the answer to the nearest *tenth*. Thus 18.47 bu. would be 18.5 bu.; 18.43 bu. would be 18.4 bu.)

UNITED STATES CROP REPORT, DECEMBER, 1921

THE SIX LEADING CORN-PRODUCING STATES	PRICE PER BUSHEL DECEMBER 1			AVERAGE	YIELD PER ACRE			AVERAGE
	1919	1920	1921		1919	1920	1921	
	cts.	cts.	cts.	cts.	bu.	bu.	bu.	bu.
Iowa	120	47	30		41.6	46.0	43.0	
Illinois	130	59	38		36.0	34.6	34.0	
Nebraska	122	41	27		26.2	33.8	28.0	
Indiana	125	59	37		37.0	40.5	36.0	
Missouri	138	64	40		27.0	32.0	30.0	
Texas	118	84	54		30.0	26.0	25.2	

9. The table on page 203 shows the rates on freight from Chicago to several cities. What per cent of first-class rates are the second-class rates for Illinois points? The third-class rates? The fourth-class rates? Does the same percentage hold for outside points?

From Chicago to	First Class Freight per 100 Pounds	Second Class Freight per 100 Pounds	Third Class Freight per 100 Pounds	Fourth Class Freight per 100 Pounds
ILLINOIS—				
Bloomington...	.60	.51	.40	.30
Cairo.....	.89	.75	.60	.45
Danville.....	.60	.51	.40	.30
East St. Louis.	.79	.67	.53	.40
Effingham.....	.73	.62	.49	.37
Galena.....	.67	.57	.45	.34
Joliet.....	.42	.36	.28	.21
Kankakee.....	.48	.41	.32	.24
La Salle.....	.54	.46	.36	.27
IOWA—				
Burlington....	.73	.62	.49	.37
Cedar Rapids..	.79	.64	.48	.35
Council Bluffs.	1.22	.99	.69	.49
Davenport....	.67	.57	.45	.34
Decorah.....	.92	.76	.61	.41
Des Moines...	.92	.73	.55	.42
Dubuque.....	.69	.59	.46	.35
Ft. Dodge.....	.99	.79	.60	.45
KANSAS—				
Concordia.....	1.91	1.59	1.18	.90
Dodge City...	2.46	2.12	1.65	1.27
Great Bend...	2.21	1.86	1.45	1.18
Leavenworth..	1.22	.99	.69	.49
Pittsburg.....	1.42	1.28	.92	.69
COLORADO—				
Craig.....	5.61	4.79	3.99	3.26
Denver.....	2.74	2.21	1.67	1.30
Durango.....	4.71	3.97	3.15	2.36
Grand Junction	3.98	3.35	2.78	2.24

10. Mr. S moves from Chicago to Council Bluffs, Iowa. Household goods are ranked as first-class freight, and his furniture weighs 4475 pounds. The railroad fare is $14.96 for one adult. Mr. S has to pay transportation for his two children, for each of whom he pays half fare, his wife, and his mother. His drayage bill for hauling goods to the station is $35.00 and the expense of crating the furniture is $87.50. What does it cost Mr. S to change his place of residence?

11. Mr. B owns a hardware store in Dubuque, Iowa. A shipment of goods from Chicago is listed on the freight bill as follows:

Articles A, B, C; wts., 124, 75, 285, second class.
Articles D, E, F, G; wts., 95, 246, 360, 155, third class.

What is the amount of the freight bill?

12. Stoves shipped from Chicago are listed third class. How much more does a hardware merchant in Craig, Colo., pay for freight on a 175-pound stove than a merchant in Pittsburg, Kans.?

13. Compute the freight on the following items: 220 pounds, first class; 325 pounds, second class; 420 pounds, third class; 550 pounds, fourth class, shipped from Chicago to East St. Louis.

14. Let the teacher assign a city for each row of pupils and dictate a series of problems like number 13. Each pupil is to compute the freight bill for his city. See which row can get the best percentage of correct answers in a given time or which row can get all answers correct in the shortest time.

15. Find totals and averages in the table of railroad accidents on page 205.

Number Killed and Injured by Railroads, 1916–1921				
Year	Passengers		Total No. of Persons	
	Killed	Injured	Killed	Injured
1916	291	8,008	10,001	196,722
1917	343	8,374	10,087	194,805
1918	519	8,082	9,286	174,575
1919	301	8,147	6,978	149,053
1920	264	8,456	6,958	168,309
1921	226	6,144	5,996	120,685
Totals				
Averages				

16. Make a comparison by means of line graphs of the number of passengers killed year by year.

17. Draw a graph on cross-ruled paper showing the variation in numbers injured year by year.

18. What is the ratio of the average number of passengers killed to the average number of all persons killed for the six-year period?

19. Use the result of Exercise 18 and an equation to find what per cent of total number killed were passengers.

20. For each year find what per cent of the total number injured were passengers.

21. A farmer had 384 bushels of wheat to sell. He sold $\frac{2}{3}$ of it at \$1.06 per bushel; the remainder at \$1.08. What was the total amount received for his crop of wheat?

22. A farmer had 360 bushels of corn to sell. He sold 25% of it at 72 cents per bushel, $33\frac{1}{3}\%$ of what was left at 68 cents per bushel, and the remainder at 75 cents per bushel. How much did he receive for his corn?

23. An automobile covers 242 miles in 12 hours. If $1\frac{1}{4}$ hours, the time consumed in getting dinner and making repairs are taken out, what was the average number of miles actually driven per hour?

24. A tank has a capacity of 7 cubic feet. How many gallons will it hold? (231 cubic inches = 1 gallon.)

25. A car of 220 bushels of potatoes was loaded in northern Maine. The potatoes cost $1.20 per bushel on the car. On the way to New York City 10 per cent of them were frostbitten. If the potatoes that were saved were sold on the car in New York City for $1.50 a bushel, what was the net profit on this transaction?

26. 100 yards are what per cent of 440 yards?

27. 220 yards are what per cent of 100 yards?

28. A laborer gets d dollars per week. Express the amount he will receive for 3 weeks; $2\frac{1}{2}$ weeks; $\frac{1}{2}$ week.

29. At the wages in Exercise 28 how will you express what the laborer gets per day? (Call a week 6 days.) For 2 days? For 3 days?

Solve Exercises 30–34 by the equation method. (See Chapter IV.)

30. A's salary in 1922 was $3762. This was $\frac{4}{5}$ of his salary in 1921. Find his salary for 1921.

31. The registration in school A was 564 pupils in 1922. This was three fourths of the registration in 1921. What was its registration in 1921?

32. When coal retails for $15.50 per ton (2000 pounds), how much should I pay for 19,000 pounds?

33. A concrete walk 72 feet long and $4\frac{1}{2}$ feet wide is laid at an expense of 48 cents per square foot. A discount of 5% is allowed if the work is paid for when completed. How much cash will be needed if payment is to be made when the work is completed?

34. The city decides to lay a concrete walk on *both* sides of a street and requires all property owners on this street to pay two thirds of the expense of laying the walk. The street is three quarters of a mile long and the walk is to be 4 feet wide. The cost of the walk is 50 cents per square foot. Determine the entire cost of the walk and the amount that the property owners must pay.

35. A owns a lot which has a frontage of 125 feet on this street. How much will he have to pay?

36. B owns a lot which has a frontage of 75 feet on the same street. How much will he have to pay?

37. A rug 12 feet 2 inches long and 9 feet 10 inches wide is placed in a room 21 feet long and 15 feet wide.

(*a*) Find the number of square feet of floor space covered by the rug (to the nearest square foot).

(*b*) Find the number of square feet of floor space in the room.

(*c*) Find the per cent of floor space covered by the rug.

(*d*) Find the per cent of floor space not covered by the rug.

38. There are six windows in the room whose dimensions are given in Exercise 37; each window is 53 inches high and 27 inches wide.

(*a*) Find to the nearest thousandth the ratio of the total window area to the floor area.

(*b*) Find the per cent the total window area is of the floor area.

39. In this room there are two open entrances; one is 6 feet 8 inches high and 6 feet wide; the other is 6 feet 8 inches high and 4 feet wide.

(*a*) Find to the nearest thousandth the ratio of the total door space to the floor area.

(*b*) Find the ratio of the total door space to the window area.

40. Cedar posts, costing 44 cents each, are to be set around the two sides and one end of a rectangular lot of land. The lot is 330 feet long and 135 feet wide and the posts are to be set 15 feet apart. Find the number of posts needed and the cost. Make a diagram.

41. Five wires are to be strung on the posts in Exercise 40. If the cost of the wire is $2\frac{1}{2}$ cents per linear foot, find the cost of the wire needed, adding 20 feet for waste.

42. How long must a strip of land 220 yards wide be to contain $1\frac{1}{2}$ acres? (Use the formula for the area of the rectangle.)

43. A lot of land 230 feet long by 70 feet wide is bought at 17 cents per square foot. Find the cost.

44. A house lot in the shape of a trapezoid is bought at 18 cents per square foot. The width of the lot is 68 feet; one of the parallel sides is 110 feet and the other 122 feet. Find the cost of the lot.

For Exercises 45–46, see Table, Exercise 8, page 202.

45. By line graphs compare the yield in bushels per acre for the three years Let 1 inch represent 10 bushels.

46. By line graphs compare the price in cents per bushel for the three years. Let 1 inch represent 50 cents.

47. The table at the top of page 209 gives the distances from New York via water to each of the following places.

Find the distance saved and the per cent of distance saved to the nearest per cent.

	Former Route (Miles)	Panama Canal (Miles)	Distance Saved (Miles)	Per Cent Saved
San Francisco . . .	13,135	5,262	?	?
Hawaii	12,800	7,000	?	?
Manila	17,800	12,000	?	?

48. The following budget gives the proper distribution, based upon thousands of careful tests, of a monthly income of $100 for each of the following items:

Food—Groceries, meats, provisions, milk, etc. . .	$31.00
Clothing and house furnishings generally	15.00
Rent, light, and heat	24.00
Car fare, life insurance, doctors, medicines	12.00
Amusements, recreation, vacation, etc	8.00
Savings and contingent funds	10.00
	$100.00

Represent this distribution by a circular graph. (See page 165.)

49. Copy the items from Exercise 48, and then determine how much should be set aside for each purpose:

(*a*) If the yearly income is $1200.

(*b*) If the yearly income is $1600.

(*c*) If the yearly income is $2400.

(*d*) If the yearly income is $3000.

50. If the family income is $2350 per year, how much should be set aside for rent, light, and heat? If the bills for light and heat average $5 per month, what is the limit of monthly rent that this family can pay?

51. How much can this family afford to spend for amusement, etc.? How much should they save?

52. The popular vote for President of the United States in November, 1920, was: Harding, 16,152,200; Cox, 9,147,350. Find the total popular vote. What per cent did Harding receive? What per cent did Cox receive?

53. The following articles are advertised to be sold at a discount of 15% from the marked price. Determine the net price in each instance:

(*a*) Coat, $18.00
(*b*) Suit, $25.00
(*c*) Overcoat, $30.00
(*d*) Overcoat, $35.00
(*e*) Overcoat, $40.00

54. An aviator traveled 290 miles in $3\frac{1}{4}$ hours. What was his average rate in miles per hour?

55. In October, 1904, a New York Central train went 3.51 miles, from Croton to Ossining, in 2 minutes. What was the average rate in miles per hour?

56. In 1897 a railroad train went from Chicago to Denver, a distance of 1,025 miles, in 18 hours 52 minutes. What was the average rate in miles per hour?

57. In a certain class 8 of every 10 pupils pass. In this class there are 35 pupils. How many pass?

58. In a certain school 3 of every 7 pupils have defective vision. If there are 840 pupils in this school, how many have defective vision?

59. Express the following as a ratio per hundred:

(*a*) $\frac{3}{4}; \frac{12}{16}; \frac{8}{32}; \frac{2.4}{4.8}; \frac{3.2}{6.4}; \frac{1.2}{3.6}$ (*b*) $\frac{1}{3}; \frac{2}{3}; \frac{1}{6}; \frac{5}{6}; \frac{6}{18}; \frac{.3}{.9}$

60. At $58.40 per 1000 ft. what will be the cost of each of the following quantities of lumber: 250 ft.? 500 ft.? 750 ft.? 375 ft.? 625 ft.? 875 ft.? Find the total cost.

61. If a 10% discount was given on the amount of the bill in the previous exercise, find the net amount of the bill.

62. At $52.00 per 1000 ft. find the cost of each of the following quantities of lumber: 210 ft.; 125 ft.; 40 ft.; 25 ft.; 22 ft. Find the total cost.

63. The following are advertised to be sold at a discount of 20% on the marked price. Determine the discount and the net price of each article.

Article	Marked Price	Discount	Net Price
a. Overcoat....	$12.50	?	?
b. Suit.....	22.50	?	?
c. Suit.....	14.50	?	?
d. Overcoat....	18.50	?	?
e. Overcoat....	22.50	?	?
f. Suit.....	27.50	?	?
g. Suit.....	32.50	?	?
h. Overcoat....	42.00	?	?
i. Overcoat....	47.50	?	?
j. Overcoat....	52.50	?	?

64. The price of an automobile is raised from $960 to $1070. What is the per cent of increase?

65. The price of an automobile is raised from $1090 to $1250. What is the per cent of increase?

66. The price of an automobile is reduced from $860 to $795. What is the per cent of reduction?

67. The Eureka School Supply Company bills the following articles to the School Board of District 25. Make out the bill and find the amount if paid in cash.

15 yds. cloth blackboard @ $2.00 per yd. less $16\frac{2}{3}\%$ discount; 1 dictionary @ $15.00 less 20%; 3 desks @ $4.50 less 10%; 1 doz. boxes crayon @ 40 cents a box

less 15%; 3 doz. erasers @ 90 cents per doz. less $16\frac{2}{3}\%$. Additional discount of 3% of net price for cash.

68. A merchant bought 4 dozen shirts at $17.50 per dozen to sell at $2.00 each. It costs him 20% of the cost of his goods to run his business (pay rent, help, etc.). He sells 28 of the shirts at the regular price and the remainder at a special 15% discount sale. How much was his net profit? What per cent of the first cost of the shirts was his profit?

69. Allowing 20% on the cost of the goods for running the business, how much net profit does a groceryman make on 20 dozen cans of corn bought at $1.50 per dozen if he sells 150 cans at the marked price of 18 cents per can and the remainder during a special $33\frac{1}{3}\%$ discount sale?

70. In a certain store it is customary to allow each clerk, in addition to his regular salary, a commission of 10 per cent on the excess of the gross amount of his sales for the week over $1000. Determine the excess over $1000 and each clerk's commission in the following list:

Clerk's Number	Amount of Sales	Excess over $1000	Commission
No. 11	$1025.75	$25.75	$2.58
No. 15	1115.75	?	?
No. 21	1505.50	?	?
No. 22	1800.00	?	?
No. 23	1985.60	?	?
Total.....	?	?	? (Check)

71. An agent's commission for selling automobiles is 15% of the selling price. Determine his commission on each of the following:

(*a*) An automobile selling for $1070.

(*b*) An automobile selling for $1250.

(*c*) An automobile selling for $1450.

72. Determine the total amount earned by an agent who sold in one month 5 automobiles of grade (*a*), 3 of grade (*b*), and 2 of grade (*c*), in Exercise 71.

73. Compute the simple interest on $1450 for 9 months at $4\frac{1}{2}$ per cent.

74. Compute the simple interest on $2560 for 11 months, 15 days at 5 per cent.

75. Find the amount due on $1250 for 8 months, 25 days at 4 per cent simple interest.

76. Find the amount due on $2275 for 7 months, 12 days at $4\frac{1}{2}$ per cent simple interest.

77. Find the amount due on $3250 for 10 months, 25 days at $6\frac{1}{2}$ per cent simple interest.

78. Mr. Brown has sixteen $100 Liberty Bonds that bear $3\frac{1}{2}\%$ interest, eight $50 Liberty Bonds that bear $4\frac{1}{4}\%$, and twenty $100 shares of railroad stock that bears $5\frac{1}{2}\%$. Interest on all these is paid semiannually. How much interest does he receive each half year?

79. Suppose Mr. Brown deposits the first half-year's interest in a savings bank at 4%. The bank adds the accumulated interest to the principal every six months and computes interest for the next six months on the whole number of dollars in the total. What is the total interest accumulation on all his investments during the first year?

80. Suppose Mr. Brown continues to invest with the savings bank his semiannual interest payments. How much interest would he receive from all his investments during the second year?

II. Drill Tables and Proficiency Tests

1. Practice giving at sight the sum of each of the following pairs of numbers.

[Suggestion. Add the tens' column first.]

12	22	33	44	55	66	77	88
11	11	11	11	11	11	11	11
23	33	44	55	66	77	21	22
22	22	22	22	22	22	13	13
14	24	23	25	26	24	34	54
13	13	13	13	13	14	14	14
25	45	35	65	32	47	71	81
14	14	14	14	23	32	28	26
45	72	35	64	57	96	63	66
32	17	33	35	52	12	42	43
46	67	93	72	46	97	72	84
38	35	21	18	23	15	41	62
71	89	78	93	77	89	98	99
29	35	26	41	44	27	72	89
67	54	38	43	76	85	96	94
36	25	19	25	31	55	52	73
38	95	27	84	49	73	37	88
29	34	16	25	36	23	18	65
75	39	94	28	86	30	80	47
25	16	54	17	66	16	55	19

Also practice giving at sight the difference between each pair of numbers.

2. Practice giving at sight the sum of each of the following pairs of fractions always expressing the answer in lowest terms.

Also practice giving the difference between each pair of fractions.

$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{7}{16}$
$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{3}{4}$
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{1}{16}$	$\frac{3}{16}$
$\frac{1}{3}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{8}{9}$
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{5}{9}$	$\frac{2}{3}$
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{6}$
$\frac{1}{2}$	$\frac{5}{6}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{7}{8}$
$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{6}$

3. Practice giving the fractional parts of the numbers on the rim of the circle in Fig. 75.

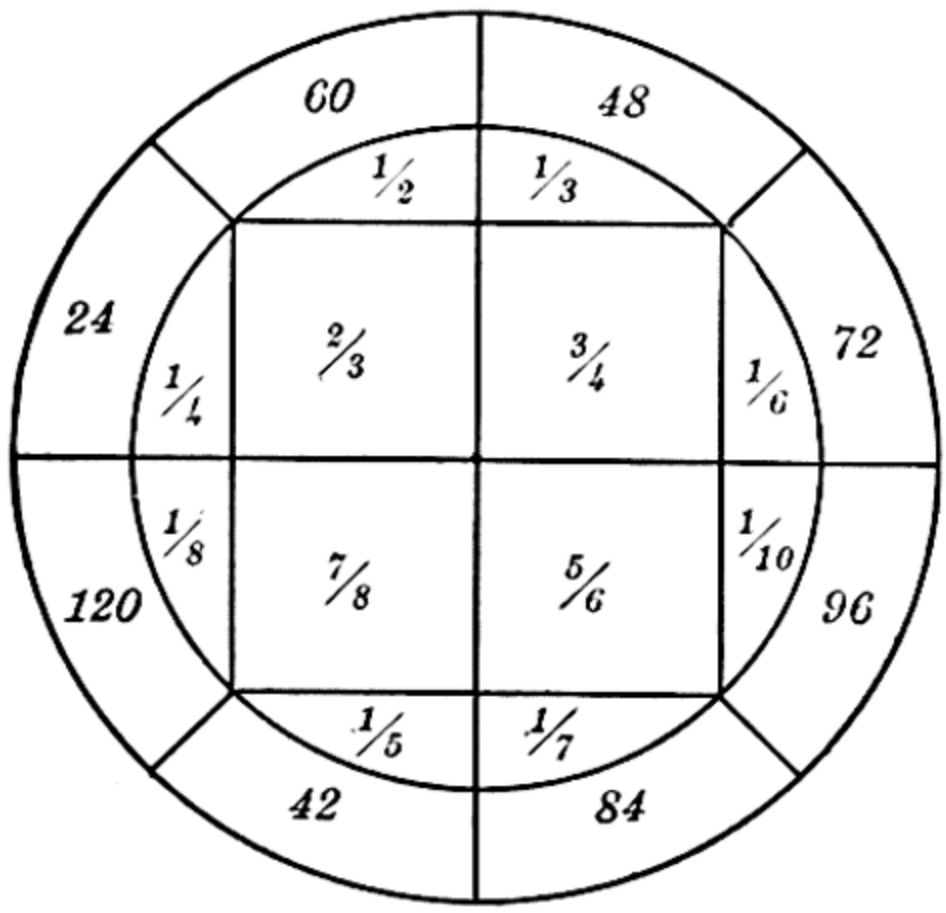


FIG. 75

4. Practice giving per cents of the numbers on the rim of the circle in Fig. 76.

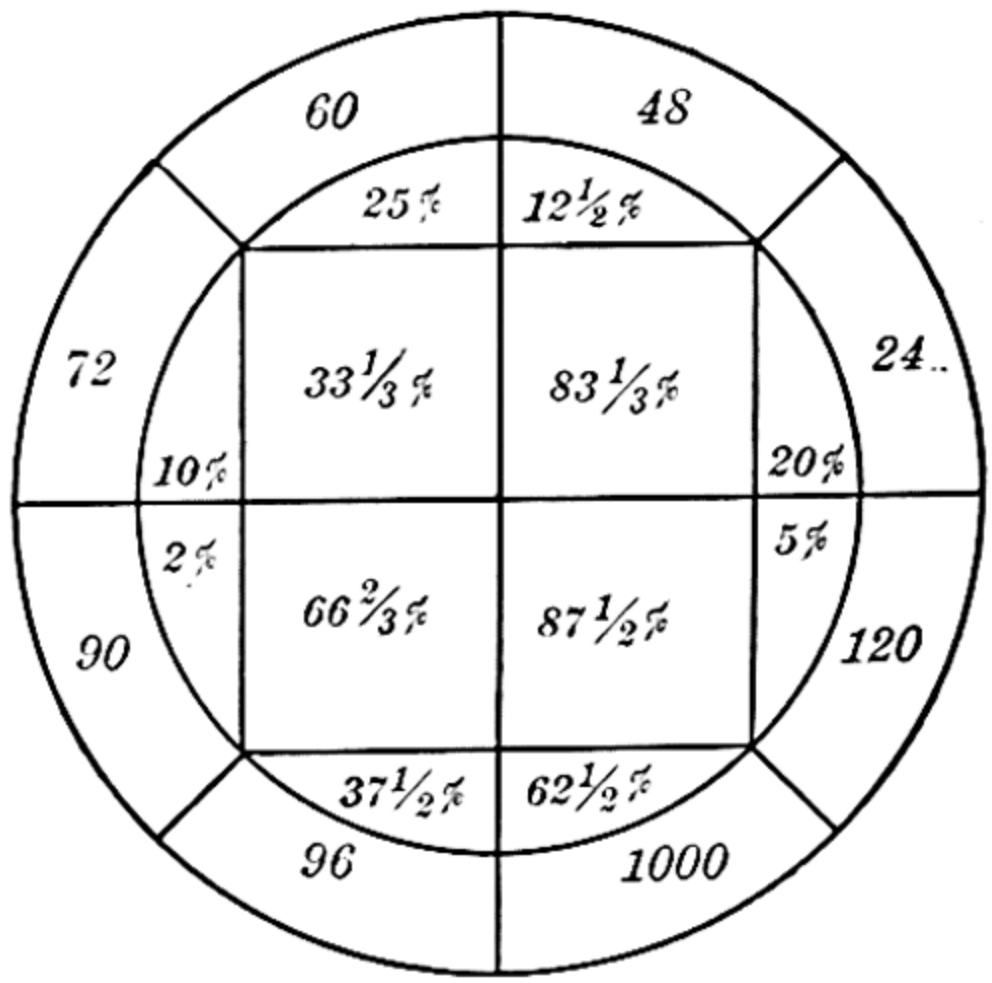


FIG. 76

5. Add as decimal fractions each of the following pairs of fractions.

$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	.6	.7	.8
.1	.2	.3	.4	.5	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
.9	$\frac{1}{5}$	$\frac{1}{5}$	.3	.4	.5	.6	.7
$\frac{1}{2}$	.1	.2	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
$\frac{2}{5}$	$\frac{2}{5}$	.5	$\frac{2}{5}$	.7	$\frac{2}{5}$	.9	.8
.1	.3	$\frac{2}{5}$	.2	$\frac{2}{5}$	.4	$\frac{2}{5}$	$\frac{2}{5}$
$\frac{3}{5}$	$\frac{3}{5}$	.7	$\frac{3}{5}$	.9	.8	$\frac{3}{5}$	$\frac{3}{5}$
.1	.3	$\frac{3}{5}$	.5	$\frac{3}{5}$	$\frac{3}{5}$	.2	.6
$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$	.9	$\frac{4}{5}$	.8	$\frac{4}{5}$	$\frac{4}{5}$
.1	.2	.6	$\frac{4}{5}$	.5	$\frac{4}{5}$	.7	.4

Find as decimal fractions the difference between each pair of fractions.

6. Add as common fractions each of the following pairs of fractions.

$\frac{1}{3}$	.75	$\frac{2}{3}$	.75	.25	$\frac{5}{6}$	$\frac{5}{6}$	.25
.25	$\frac{1}{3}$	.25	$\frac{2}{3}$	$\frac{1}{6}$	.25	.75	$\frac{1}{7}$
.25	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	.75	.75	.75	$\frac{7}{8}$
$\frac{1}{8}$	.25	.25	.25	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	.75

Find also the difference between each pair of fractions.

7. The following table shows the average prices received by the farmer for certain products. The prices are expressed in cents per bushel.

	Wheat	Corn	Oats	Rye	Potatoes	Apples
1916	102.8	62.1	39.1	85.3	70.6	79.7
1917	150.3	90.0	51.4	118.5	147.3	101.1
1918	201.9	134.8	73.9	170.3	121.0	128.8
1919	204.8	144.7	70.8	150.7	116.1	147.7
1920	231.8	140.4	78.2	152.3	178.6	213.8
1921	149.2	66.7	45.6	124.7	105.6	118.6
1922	93.3	43.4	31.0	69.6	108.6	180.6
Av.						

(*a*) Find the average of the prices of each product for the seven-year period.

(*b*) The price in 1922 of each product is what per cent of the average of the prices of that product?

(*c*) Solve, at the dictation of your teacher, problems like this: How much was received for 825 bushels of wheat at the average price in 1921?

(*d*) Draw the six graphs showing the variation in prices of all the products. Compare the graphs.

Supplement to Table I of Chapter I

2	1	6	3	7	7	5	2	8	9	2	9	3	1	4
4	8	5	9	0	6	4	2	3	1	8	5	3	7	6
4	3	9	1	0	1	2	7	8	4	8	3	2	1	6
7	8	6	5	2	6	5	3	8	9	0	5	7	9	4
3	0	5	7	8	9	8	7	6	4	2	6	7	1	4
2	1	9	4	6	3	1	7	2	5	3	9	5	1	8
8	3	6	0	9	5	0	7	1	2	8	6	7	9	5
2	7	1	5	4	3	6	8	4	9	4	3	1	9	2
3	6	1	8	0	3	5	4	7	3	1	4	5	8	9
4	7	2	5	9	1	9	2	5	8	3	4	6	7	2
2	0	3	9	5	0	2	9	4	5	8	4	7	6	5
1	4	6	8	7	3	6	7	8	1	9	3	2	6	1

Supplement to Table IV of Chapter I

258	369	167	489	257	168	357	248
726	538	802	535	816	725	608	424
852	741	963	732	864	842	971	732
864	753	831	642	853	764	942	863
204	613	825	734	917	703	827	516
137	369	246	358	137	279	358	247
826	308	917	735	839	333	444	704
842	975	742	874	531	974	864	555
666	279	136	247	358	135	479	234

Supplement to Table IX of Chapter VII

$87\frac{1}{2}\%$ of 88	75 % of 88	$16\frac{2}{3}\%$ of 72
$83\frac{1}{3}\%$ of 18	$12\frac{1}{2}\%$ of 48	30 % of 35
10 % of 34	20 % of 75	60 % of 20
50 % of 90	40 % of 25	$66\frac{2}{3}\%$ of 33
25 % of 38	$33\frac{1}{3}\%$ of 93	$37\frac{1}{2}\%$ of 40
$37\frac{1}{2}\%$ of 64	$66\frac{2}{3}\%$ of 18	$62\frac{1}{2}\%$ of 64
$66\frac{2}{3}\%$ of 48	30 % of 60	$37\frac{1}{2}\%$ of 24
75 % of 36	$66\frac{2}{3}\%$ of 78	25 % of 52
40 % of 45	$37\frac{1}{2}\%$ of 48	50 % of 92
$33\frac{1}{3}\%$ of 33	60 % of 20	10 % of 68
$12\frac{1}{2}\%$ of 36	$62\frac{1}{2}\%$ of 56	$83\frac{1}{3}\%$ of 48
20 % of 70	$16\frac{2}{3}\%$ of 90	$87\frac{1}{2}\%$ of 32

Supplement to Table X of Chapter VII

5 % of \$14.00	4 % of \$17.50	30 % of \$18.50
$16\frac{2}{3}\%$ of 28.20	$2\frac{1}{2}\%$ of 16.00	5 % of 16.20
20 % of 11.25	$12\frac{1}{2}\%$ of 16.40	40 % of 13.50
4 % of 12.50	8 % of 7.50	6 % of 13.00
$33\frac{1}{3}\%$ of 12.60	40 % of 24.50	4 % of 14.50
30 % of 15.50	2 % of 11.50	2 % of 37.50
6 % of 22.00	20 % of 12.75	$2\frac{1}{2}\%$ of 16.00
8 % of 10.50	$37\frac{1}{2}\%$ of 32.00	$16\frac{2}{3}\%$ of 15.30
$12\frac{1}{2}\%$ of \$16.80	$16\frac{2}{3}\%$ of \$24.60	$33\frac{1}{3}\%$ of \$18.30
$2\frac{1}{2}\%$ of 12.00	10 % of 16.50	$37\frac{1}{2}\%$ of 40.80
15 % of 32.00	30 % of 17.00	8 % of 14.50
2 % of 12.50	3 % of 19.00	$12\frac{1}{2}\%$ of 20.40
$37\frac{1}{2}\%$ of 24.00	$33\frac{1}{3}\%$ of 12.90	3 % of 23.00
3 % of 13.00	5 % of 22.00	10 % of 27.50
10 % of 48.50	6 % of 20.50	20 % of 23.50
40 % of 22.50	15 % of 28.00	15 % of 22.00

Allow yourself twenty minutes for each of the following tests. Try to get all answers correct.

I

1. Add:

2694
3829
9605
2847
3942
8275
6666
8234

2. Find the products: (*a*) 9745 × 247 (*b*) \$483.75 × 9.2

3. Find the quotients:

(*a*) $75\overline{)17700}$ (*b*) $28\overline{)4096}$

4. Write the sums indicated below:

$\frac{1}{2} + \frac{3}{4} + \frac{1}{8} = ?$ $4\frac{1}{2} + 3\frac{2}{3} + 7\frac{5}{6} = ?$

$\frac{2}{3} + \frac{1}{6} + \frac{1}{12} = ?$ $7\frac{3}{16} + 8\frac{1}{2} + 2\frac{3}{8} = ?$

5. Compute the following per cents:

$16\frac{2}{3}\%$ of 75 60% of 35

$87\frac{1}{2}\%$ of 40 25% of 58

II

1. Subtract:

82740	68219	92481	70145
38527	43297	60473	22354

2. Add as decimals:

$4\frac{1}{2}$	17.25	$3\frac{2}{5}$	.75
3.2	$13\frac{3}{4}$	$4\frac{3}{8}$	$4\frac{1}{5}$

3. Find these products: $8\frac{3}{4} \times 27\frac{1}{2}$; $12\frac{2}{5} \times 10\frac{2}{3}$.

4. Solve and check this equation: $3a + 2a + 6a = 13.2$

5. Compute the net prices on the following:

4 Plows @ \$17.50 less 20%
8 Harrows @ \$32.25 less $16\frac{2}{3}\%$
2 Wagons @ \$97.50 less $33\frac{1}{3}\%$

REVIEW TEST I

Check your work carefully as you solve the problems of this test. Strive for a perfect score. Allow yourself time enough to do the work well and record your time. You should finish in *one* hour.

1. Copy, add, and average the following numbers: 629, 481, 384, 679, 425, 326, 509, 675, 768.

2. Divide 867.25 by 79.2 to two decimal places.

3. Find the sum of $4\frac{1}{2}$, $12\frac{2}{3}$, $11\frac{5}{6}$, $18\frac{1}{12}$.

4. Divide $23\frac{3}{4}$ by $7\frac{3}{5}$.

5. What is the net price of an article marked at \$24 less discount of 15%?

6. Compute the simple interest on \$325 for 4 months at 6%.

7. In Fig. 77 the side of the square is 10 inches:

(*a*) Find the perimeter of the square; of the circle.

(*b*) Find the area of the square; of the circle.

(*c*) Find the area between the boundaries of the circle and the square.

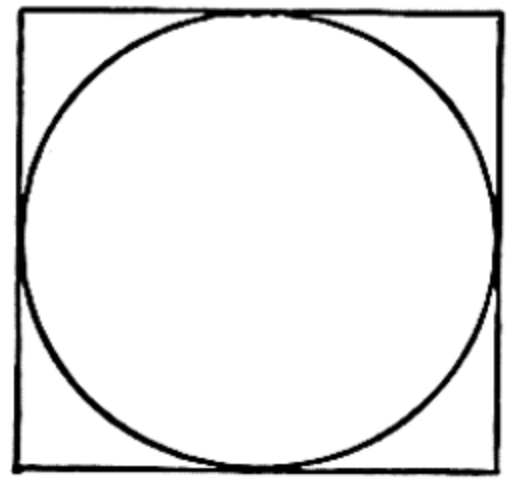


FIG. 77

8. Draw to the scale of 100′ to 1″ a rectangular lot 50′ by 140′. From a point in one of the longer sides 60′ from the end draw lines to both ends of the opposite side, forming with that side a triangle. What is the ratio of the area of this triangular plot to the area of the whole lot?

9. Find the number whose ratio to 180 is $\frac{11}{15}$.

10. Two brothers weed the onions in their father's garden for \$1.35, sharing in proportion to the amount of work done. The younger weeds half as many rows as his brother. How much money should each receive?

REVIEW TEST II

Follow the instructions given on the preceding page for Review Test I.

1. Subtract:

92648	\$495.08	269.240	80327
29075	329.17	83.326	35344

2. Find the sum of the products: 87.2×6.5, 25.6×18, and 72.8×10.4.

3. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} - \frac{1}{6} + \frac{5}{12} = ?$

4. What per cent of 30 is 24? 75 is what per cent of 90?

5. Multiply $14\frac{3}{5}$ by $17\frac{1}{2}$ and divide the product by $5\frac{1}{3}$.

6. Compute these per cents:

$12\frac{1}{2}\%$ of 48	40% of 55	$37\frac{1}{2}\%$ of 24
$66\frac{2}{3}\%$ of 72	$83\frac{1}{3}\%$ of 60	75% of 56

7. Suppose the rectangle in Fig. 78 is 20″ long and 10″ wide. The circles are drawn with the centers at the vertices of the right angles. How much longer is the perimeter of the rectangle than the sum of the four circle arcs?

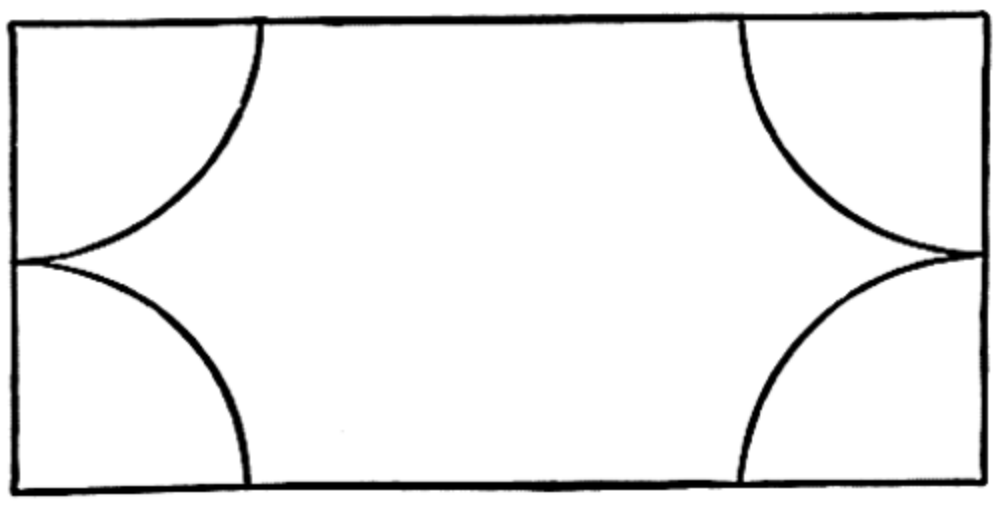


FIG. 78

8. The area of a triangle is 100 square inches and the base is 25 inches. Find the altitude.

9. Draw a graph picturing the change of temperature for the following ten successive hourly readings: 75°, 79°, 81°, 82°, 76°, 72°, 78°, 80°, 85°, 82°.

10. Find the interest on \$750 for 7 months, 15 days at $4\frac{1}{2}\%$.

APPENDIX

REFERENCE TABLES

1. Linear Measure

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.).
320 rods, or 5280 feet = 1 mile (mi.).
6 feet = 1 fathom.
1.151 miles = 1 knot.

2. Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.).
160 square rods = 1 acre (A.).
640 acres = 1 square mile.
36 square miles = 1 township.

3. Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).
16 cubic feet = 1 cord foot (cd. ft.).
128 cubic feet = 1 cord (cd.).

4. Dry Measure

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

5. Liquid Measure

4 gills (gi.) = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon (gal.).
63 gallons = 1 hogshead.

6. Avoirdupois Weight

16 ounces (oz.) = 1 pound (lb.).
2000 pounds = 1 ton (T.).
32 pounds = 1 bushel of oats.
48 pounds = 1 bushel of barley.
56 pounds = 1 bushel of corn or rye.
60 pounds = 1 bushel of wheat or potatoes.
196 pounds = 1 barrel (bbl.) of flour.
2240 pounds = 1 long ton of coal.

7. United States Money

10 mills (m.) = 1 cent (ct. or ¢).
10 cents = 1 dime (di.).
10 dimes = 1 dollar ($).
100 cents = 1 dollar.
10 dollars = 1 eagle.

8. Time

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
7 days = 1 week (wk.).
365 days = 1 common year (yr.).
366 days = 1 leap year.
12 months = 1 year.
360 days = 1 commercial year.
100 years = 1 century.

9. Counting

12 units = 1 dozen (doz.).
12 dozen = 1 gross (gro.).
24 sheets of paper = 1 quire.
500 sheets = 1 ream.

10. Circular Measure

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
360 degrees = 1 circumference.
$69\frac{1}{6}$ miles = 1 degree of latitude.

EQUIVALENTS

1 bushel = $\frac{5}{4}$ cu. ft., or 2150.42 cu. in.
1 gallon = 231 cu. in.
1 cu. ft. = $7\frac{1}{2}$ gal. (approx.).
1 ton of hay = 500 cu. ft. (approx.).
1 ton of coal = 35 cu. ft. (approx.).

FORMULAS

I. NOTATION USED IN FORMULAS

A = area.
b, b_1, b_2 = bases.
h = height or altitude.
s = side.
c = circumference
d = diameter.
r = radius.
π = 3.1416

II. FORMULAS

1. Area of a rectangle, $A = bh$, p. 132.
2. Area of a square, $A = s^2$, p. 140.
3. Area of a parallelogram, $A = bh$, p. 144.
4. Area of a triangle, $A = \frac{1}{2}bh$, p. 146.
5. Area of a trapezoid, $A = \frac{1}{2}(b_1 + b_2)h$, p. 151.
6. Circumference of a circle, $c = \pi d$ or $2\,\pi r$, p. 160.
7. Area of a circle, $A = \pi r^2$, p. 161.

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